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**Optimal Pricing for a Firm that Leases New Products and Sells
Remanufactured Ones**

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Abstract

In this paper we consider a firm that leases a single type of product and also sells remanufactured versions of this product that become available at the end of their lease periods. We assume that when the quantity of end-of-lease items in stock is not sufficient to meet the demand for remanufactured products, the firm has the option to purchase additional cores from a third-party core supplier at a certain unit cost. Customers have a lower willingness-to-pay for the remanufactured product and their preference is modeled using a maximum utility approach. We develop a multi-period model to determine the selling price of remanufactured products and the periodic lease payments of new products. We investigate the effect of key product characteristics such as deterioration in age and unit cost of purchasing cores from a third-party supplier, and key target market characteristics such as relative willingness-to-pay for buying a remanufactured product and relative willingness-to-pay for leasing a new product. The resulting problem is a nonlinear optimization problem which is solved by a variant of Nelder-Mead simplex search method.

1 Introduction

Remanufacturing is the process of disassembling used products, inspecting and repairing their components and using these in manufacturing new products. A product is considered remanufactured if its primary components come from a used product. Recently, remanufacturing has been receiving growing attention for various reasons such as consumer awareness, environmental concerns, economical benefit, and legislative pressure. Remanufactured products include photocopiers, computers, telecommunication equipment, cellular phones, automotive parts, office furniture, and tires. One of the major issues faced by the firms involved in remanufacturing is taking back used products before the end of their useful life so that some revenue can be generated by remanufacturing or reusing them. Another concern is the uncertainty in the quantity, timing and quality of returned products. At this point, leasing turns out to be a viable strategy that helps to better manage the return process.

Leasing is a widely used business strategy in United States, with 80 per cent of all US companies leasing some or all of their equipment and an estimated 226 billion worth of equipment leased in the US according to a research done by Equipment Leasing Association (ELA) in 1999. Leasing helps to the firm in getting a consistent flow of used products for remanufacturing, which in fact reduces the uncertainty in the quantity and timing for returns. Furthermore, it has a positive impact on better forecasting the quality of returns at the end of their lease time because of the periodic maintenance activities performed by the firm. As a consequence, leasing allows firms to control the quality, quantity and timing of product returns, which is a primary concern of many remanufacturing initiatives.

Besides the benefits of leasing for the manufacturer, consumers have also advantages when they lease products instead of buying them. As such, they pay for the service provided by the product rather than for the product itself. Note that only a portion of the product's price is paid, which corresponds to the proportion "used up" during the lease period. Moreover, leasing products like computers makes it easier for customers to sooner upgrade to the newest technology. Certainly, product characteristics affect the viability of leases. For example, nondurable products are not suitable for leasing since little value remains at the end of the

lease period to be captured, so that no product recovery becomes possible except recycling that is considered to be the least desirable recovery form.

In this paper, we consider a firm leasing one type of new products, and selling remanufactured versions of these products. Hence, we assume that the product is durable and remanufacturable. Our main objective is to determine in a multi-period setting the best prices so as the profit of the firm is maximized. Customers in the market are assumed to be heterogeneous in their willingness-to-pay and they perceive the value of remanufactured products less than the new products. Customers are assumed to be grouped in different segments where each segment corresponds to customers that are interested in a certain lease period. Customer preference in each segment is modeled using a maximum utility type approach in which customers make their choice of leasing a new product, buying a remanufactured product, or buying nothing based on the utility they derive from each choice. Leased products return at the end of the lease agreement. This implies that at the beginning of each period we exactly know how many cores will be returned. Therefore, the supply of used products depends on past lease volumes of new products. If returned products in stock are not sufficient to meet demand for remanufactured products in any period, the manufacturer may obtain extra used products from a third-party core supplier. Some of the questions we address are:

- What are the optimal prices in each period?
- How do the factors such as deterioration in age and cost of acquiring used remanufacturable products from the third-party core supplier influence the optimal price in each period?
- How does customers' perception of remanufactured products with respect to new products affect the optimal pricing strategy?

The remainder of the paper is organized as follows: Section 2 includes a literature review about leasing, and pricing-oriented papers within the context of remanufacturing. Problem

description and model formulation are given in Section 3. Section 4 includes the solution procedure. Experimental results are presented in Section 5. We conclude in Section 6.

2 Literature Review

We provide a brief literature review on leasing and pricing within the remanufacturing context. Leasing as a means of transaction is playing an increasingly important role in marketing durable goods. On the other hand, leasing is also becoming a pervasive phenomenon in our ordinary life. For instance, many durable goods that are traditionally sold to consumers can now be leased too. The spectrum of leased durable goods is rapidly expanding. Examples of these include such daily necessities as cars, furniture, computers, and other electronic appliances (Huang and Yang, 2002).

Fishbein et al. (2000) have examined the practice of leasing products, rather than selling them, as a strategy for increasing resource productivity, particularly by preventing waste generation and moving to a pattern of closed-loop materials use. They mention many companies that successfully acquire products through leasing and remanufacture returned products. One of the well known examples is Xerox, whose goal is to be the “leader in the global document market” with its document-processing products, systems and services. There are similar examples from industry sectors such as carpet (e.g., Interface Inc.), computers (e.g., Compaq, Dell, Gateway, and IBM).

Desai and Purohit (1998) analyze the problems associated with marketing a durable through leases and sales. Their goal is to understand the strategic issues associated with concurrently leasing and selling a product and determine the conditions under which this concurrent strategy is optimal. They model a market in which both leases and sales are allowed, and a durable product is marketed in a two-period structure. Desai and Purohit (1999) also examine competition in a duopoly; they investigate a firm’s rationale in choosing an optimal mix of leasing and selling and to understand how it is affected by the nature of competition in the market and the embedded quality in the product. They develop a two-period model in which consumers are indifferent between buying and leasing a durable

product. They find that a competitive environment forces firms to adopt strategies where they only sell their products or use a combination of leasing and selling.

The model proposed by Sharma (2004) allows electronic equipment leasing companies to simultaneously make optimal decisions about lease lengths, product flows and end-of-life product disposal. Pricing is not taken into account in this model since pricing is a strategic decision that is affected by many other factors like market competition, sales and marketing strategies, economic and political conditions, etc. The model is deterministic, but the examination of uncertainty in problem parameters has been made by solving multiple scenarios of this model.

There is a growing literature in operations management that combines remanufacturing, pricing of new and remanufactured products, competition and marketing. These studies try to determine the optimal selling prices of remanufactured and new products to maximize the profit of the company. Groenevelt and Majumder (2001) develop a two-period model to examine the effect of competition in remanufacturing considering one OEM and a local remanufacturer. In the first period, only the OEM manufactures and sells items. In the second period, a fraction of these items are returned for remanufacturing. However, some returned items are acquired by the local remanufacturer. Thus, competition exists in the second period for remanufacturing returned items and selling them.

The effect of competition on recovery strategies has also examined by Ferguson and Toktay (2004) with some differences. The choice of a manufacturer whether or not to recapture the value in their end-of-life products through remanufacturing is driven by two concerns: cost and internal cannibalization. On the cost side, the cost to remanufacture plus the fixed cost needed to establish a remanufacturing operation may be too high to enter remanufacturing. However, even if the remanufactured product is independently profitable, firms may ignore this option due to concerns about cannibalization. Two-entry deterrent strategies are developed by Ferguson and Toktay (2004): remanufacturing and preemptive collection. Preemptive collection is a strategy to discourage competition so that manufacturer collects part or all of the items, but does not recover the residual value of the used product. They

find that a firm may prefer to remanufacture or preemptive collection to deter entry, even when the firm would not have chosen to do so under a pure monopoly environment.

The study of Debo et al. (2005) has important insights since it is the first study that addresses the integrated market segmentation and production technology choice problem in a remanufacturing setting where the supply of used products that can be remanufactured depends on the past sales volumes of new products and the level of remanufacturability. Previous papers in the literature take the remanufacturability level as exogenously determined while this paper introduces the level of remanufacturability as a key variable. They solve the joint pricing and production technology selection problem faced by a manufacturer that considers introducing a remanufacturable product in a market that consists of heterogeneous customers. In the model, production technology selection determines the remanufacturability level. The customer preferences are explained through a maximum utility approach. Moreover, they try to answer the question how competition with independent remanufacturers should be taken into account when determining the remanufacturability level, because manufacturer can control the remanufacturability level, and therefore control the supply of remanufactured products to independent remanufacturers.

Mitra (2007) notes that because of skepticism about the quality of remanufactured products, not all remanufactured products would be sold, and also there could be different quality levels of recovered products, which would draw different prices in the secondary market. The author discusses two pricing models in the context of recycled cellular phones in India to maximize the expected revenue from the recovered products. Two quality levels are taken into account, namely remanufactured products which are as good as new and refurbished products which are of lower quality. The objective of the paper is to determine prices of the remanufactured and refurbished products such that the total revenue is maximized.

Ferrer and Swaminathan (2006) analyze a model where remanufactured and new products are not distinguishable to the customer. They analyze two-period and multi-period scenarios where the manufacturer only produces the new product in the first period, but has the option of making new and remanufactured products in subsequent periods. Next, they

focus on the duopoly environment where an independent remanufacturer may obtain cores to sell remanufactured products in future periods. They observe that as the marginal cost of remanufacturing decreases, the value of making new products in the first period increases, and the value of making new products in future periods decreases. In other words, if remanufacturing is very profitable, the firm tries to increase the available cores for remanufacturing later. This behavior does not change, whether the OEM is a monopolist or not, operating with any planning horizon.

3 Model Formulation

3.1 Problem Description

We study the optimal pricing strategy of a profit-maximizing firm leasing new products, and selling remanufactured products. We assume that the product is both durable and remanufacturable, and the manufacturer offers only one type of product rather than a diversified product line. The product must undergo a remanufacturing operation before being sold as a remanufactured product. We also assume that used products can be remanufactured only once, and leasing is not an option for remanufactured products. Another assumption is that the duration of lease agreements, L , cannot exceed the life cycle of the product. Here, we allow the product to have a shorter residence time (duration of one use of the product by a lessee) than the life cycle M of the product.

The durability of the product also suggests that second-hand markets may play an important role, because secondary market prices and equipment availability can impact new product sales and pricing, but leasing also provides manufacturers with greater control over the resale market. Therefore, we do not take into account the availability of the second-hand market. While a product is in use, it deteriorates (depreciates) by rate d_m where m is the lease period expressed in years. For a customer who leases a new product, the perceived residual value of the product at the end of m periods is assumed to be $p_n(1 - d_m)$ where p_n is the selling price of the new product. Note that as d_m increases, residual value decreases and

periodic payments increase. In other words, a higher level of depreciation rate requires higher leasing payments. In this setting, it is important to note that these rates are exogenous to the system.

We assume that customers differ in their reservation price for purchasing a new product, i.e., the maximum amount of money they would be willing to pay for having a new product. In order to model the heterogeneity of the customers, the reservation price θ of customers is defined as a random variable which is assumed to be distributed uniformly in the interval $[0, 1]$. The uniform assumption represents a large degree of variability within customer market and has become a standard assumption in the marketing literature (e.g., Debo et al., 2005; Debo et al., 2006). Let $f(\theta)$ and $F(\theta)$ be the density and cumulative distribution function of θ , respectively. A customer's reservation price for leasing a new product for m periods can be approximated as a fraction l_m of θ . If a product is leased throughout its life cycle, i.e., $m = L$, then $l_m = 1$.

Typically, customers value remanufactured products less than new products. Therefore, we assume that customers' reservation price for a remanufactured product is a fraction δ of their reservation price for a new product. Note that $\delta = 0$ implies that customers are not willing to pay anything for the remanufactured product, while $\delta = 1$ means that new and remanufactured items are considered as perfect substitutes for each other and customers are willing to pay the same amount. Since most products fall between the two extremes, we assume $0 < \delta < 1$. As a result, a customer with reservation price of θ for a new product is willing to pay at most $\delta\theta$ for a remanufactured product.

The potential market size is normalized so that it is equal to one. Furthermore, we assume that customers can be divided into groups where each group consists of customers who are willing to lease a new product for m periods. The fraction of each group is known and denoted by α_m . Clearly, $\sum_{m=1}^L \alpha_m = 1$. We model customers' decisions using the notion of consumer surplus. It is the difference between the reservation price of a customer and the price of the product. Consumer surplus measures the welfare that customers derive from the consumption of goods and services. Let p_n and p_r denote the selling prices of new

and remanufactured products, respectively. The consumer surplus of a customer associated with leasing a new product for m periods, buying a remanufactured product, and choosing to buy nothing is given by $l_m\theta - PV(p_m)$, $\delta\theta - p_r$, and zero, respectively. Here, $PV(p_m)$ represents the present value of all monthly payments, and p_m denotes the monthly payment of a customer who leases a new product for m periods. For example, when a product is leased for two periods (years), $m = 2$ and there will be 24 monthly payments p_2 . The monthly payments are computed as

$$p_m = p_n \left(\frac{d_m}{12m} + (2 - d_m) MF \right) \quad m = 1, \dots, L. \quad (1)$$

MF in expression (1) is called money factor which is calculated as $i/2400$ with i denoting annual interest rate. The present value $PV(p_m)$ of lease payments is given by

$$PV(p_m) = \frac{\beta_m (1 - \beta_m^{12m})}{1 - \beta_m} p_n \left(\frac{d_m}{12m} + (2 - d_m) MF \right) \quad m = 1, \dots, L. \quad (2)$$

The term $\frac{\beta_m (1 - \beta_m^{12m})}{1 - \beta_m}$ is the coefficient of the present value of equal payments and β_m is the monthly discount factor in the interval $[0, 1]$ expressed as

$$\beta_m = \frac{1}{1 + \frac{i}{12}} \quad m = 1, \dots, L. \quad (3)$$

As a result, the set of customers who would prefer to lease a new product for m periods is given by

$$\Omega_{n,m}(\mathbf{p}) = \{\theta \in [0, 1] : l_m\theta - PV(p_m) \geq \delta\theta - p_r\} \quad m = 1, \dots, L. \quad (4)$$

However, not all customers in $\Omega_{n,m}$ make a lease agreement. The consumer surplus $l_m\theta - PV(p_m)$ must also be positive, otherwise the option of neither leasing a new product nor buying a remanufactured product with zero consumer surplus turns out to be better. The set of customers who would prefer to buy a remanufactured product instead of leasing

a new product for m periods is denoted by $\Omega_{r,m}(\mathbf{p})$ and given as

$$\Omega_{r,m}(\mathbf{p}) = \{\theta \in [0, 1] : l_m\theta - PV(p_m) \leq \delta\theta - p_r\} \quad m = 1, \dots, L. \quad (5)$$

Let q_n and q_r denote the volume of customers who lease new products and purchase remanufactured products, respectively, and define $\mathbf{q} = (q_n, q_r)$. Recall that α_m denotes the size of the customer segment in the market who would prefer to lease a new product for m periods or buy a remanufactured product. Then,

$$q_n = \sum_{m=1}^L \alpha_m \int_{\Omega_{n,m}(\mathbf{p})} dF(\theta), \quad (6)$$

and

$$q_r = \sum_{m=1}^L \alpha_m \int_{\Omega_{r,m}(\mathbf{p})} dF(\theta). \quad (7)$$

Note that the total volume of customers q_n who lease a new product is expressed as the sum of customers in each segment m who lease new products, i.e., $q_n = \sum_{m=1}^L q_m$ where $q_m = \alpha_m \int_{\Omega_{n,m}(\mathbf{p})} dF(\theta)$. The volume of customers in segment m who do not buy anything is given by q_{0m} .

Once the prices p_n and p_r are announced, the volume of customers who lease a new product or do not prefer either leasing or purchasing can be determined. It turns out that depending on the sign of the term $(l_m - \delta)$, we have different cases. If $(l_m - \delta)$ is positive, customers in segment m with $\theta \geq A_1$ lease a new product where

$$A_1 = \frac{p_n S_m - p_r}{l_m - \delta}, \quad (8)$$

and

$$S_m = \frac{\beta_m (1 - \beta_m^{12m})}{1 - \beta_m} \left(\frac{d_m}{12m} + (2 - d_m) MF \right) \quad m = 1, \dots, L. \quad (9)$$

Table 1 contains all possible cases for $l_m - \delta > 0$. In this setting, there are customers who are indifferent between buying nothing and a remanufactured product (having $\theta_l(\mathbf{p})$) and

indifferent between preferring a remanufactured product and new product (having $\theta_h(\mathbf{p})$) in each period. Therefore, $\Omega_{r,m}(\mathbf{p}) = [\theta_l(\mathbf{p}), \theta_h(\mathbf{p})]$ and $\Omega_{n,m}(\mathbf{p}) = [\theta_h(\mathbf{p}), 1]$.

If $(l_m - \delta)$ is negative, we investigate the volume of customers who lease a new product for m periods according to $\theta \leq A_2$ where

$$A_2 = \frac{p_r - p_n S_m}{\delta - l_m}. \quad (10)$$

Table 2 includes all the possible case provided that $l_m - \delta < 0$. There are customers who are indifferent between buying nothing and a new product (having $\theta_l(\mathbf{p})$) and indifferent between leasing a new product and remanufactured product (having $\theta_h(\mathbf{p})$) in each period. Therefore, $\Omega_{n,m}(\mathbf{p}) = [\theta_l(\mathbf{p}), \theta_h(\mathbf{p})]$ and $\Omega_{r,m}(\mathbf{p}) = [\theta_h(\mathbf{p}), 1]$.

If $l_m - \delta = 0$, the volume of customers who prefer leasing a new product or nothing is given in Table 3.

In our model, remanufactured product sales depend on the availability of returning used products in each period, but we allow the manufacturer to obtain used products from the third-party core supplier when the resulting demand is greater than the available inventory. Since remanufactured products are sold and cannot be remanufactured a second time, we assume that they do not return to the manufacturer, and, perhaps, they are disposed of by the user at the end of the usage. Let R_t denote the volume of leased products that return from the market at the beginning of period t . It can be written as

$$R_t = \sum_{m=1}^{\min(L,t)} q_{m,t-m}, \quad (11)$$

where $R_1 = 0$. The indices m and $t - m$ denote the lease duration and the beginning of the lease period of a new product, respectively. The product leased in time $t - m$ becomes available in period t .

Recall that all leased products have to return at the end of the lease term which results in a known stock of used remanufacturable products available in period t . We assume that used products are available for remanufacturing as soon as they return. Let I_t be the volume

Table 1: The volume of customers leasing a new product or no product under $l_m - \delta > 0$

Case		q_m	q_{0m}
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	$\alpha_m(1 - \max\{A_1, \frac{p_n S_m}{l_m}\})$	$\alpha_m(\min\{A_1, \frac{p_n S_m}{l_m}, \frac{p_r}{\delta}\})$
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	0	α_m
$l_m - \delta > 0$	$0 \leq A_1 \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$
$l_m - \delta > 0$	$A_1 < 0$	$\alpha_m(1 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$
$l_m - \delta > 0$	$A_1 < 0$	0	α_m
$l_m - \delta > 0$	$A_1 > 1$	0	α_m
$l_m - \delta > 0$	$A_1 > 1$	0	$\alpha_m \frac{p_r}{\delta}$

Table 2: The volume of customers leasing a new product or no product under $l_m - \delta < 0$

Case		q_m	q_{0m}
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	0	α_m
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$\alpha_m(A_2 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$\alpha_m(A_2 - \frac{p_n S_m}{l_m})$	$\alpha_m(\frac{p_n S_m}{l_m} + 1 - A_2)$
$l_m - \delta < 0$	$0 \leq A_2 \leq 1$	$\alpha_m(A_2 - \frac{p_n S_m}{l_m})$	$\alpha_m(\frac{p_n S_m}{l_m} + (\frac{p_r}{\delta} - A_2))$
$l_m - \delta < 0$	$A_2 < 0$	0	α_m
$l_m - \delta < 0$	$A_2 < 0$	0	$\alpha_m \frac{p_r}{\delta}$
$l_m - \delta < 0$	$A_2 > 1$	0	α_m
$l_m - \delta < 0$	$A_2 > 1$	$\alpha_m(1 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$

Table 3: The volume of customers leasing a new product or no product under $l_m - \delta = 0$

Case			q_m	q_{0m}
$l_m - \delta = 0$	$p_n S_m - p_r \leq 0$		$\alpha_m (1 - \frac{p_n S_m}{l_m})$	$\alpha_m \frac{p_n S_m}{l_m}$
$l_m - \delta = 0$	$p_n S_m - p_r > 0$	$\frac{p_r}{\delta} > 1$	0	α_m
$l_m - \delta = 0$	$p_n S_m - p_r > 0$	$\frac{p_r}{\delta} \leq 1$	0	$\alpha_m \frac{p_r}{\delta}$

of used products that remain in stock at the end of period t . Then,

$$I_t = I_{t-1} + R_t - q_{r,t}. \quad (12)$$

The inventory of remanufacturable products at the end of the current period is equal to the inventory at the end of the last period, plus the supply of remanufacturable products that become available at the beginning of period t , minus the amount remanufactured. If the prices in period t are chosen such that the resulting demand $q_{r,t}$ for remanufactured products is greater than the available inventory ($I_{t-1} + R_t$), the shortage $\Delta_{r,t}$ in used products is obtained from the third-party core supplier. $\Delta_{r,t}$ is given by

$$\Delta_{r,t} = \max(q_{r,t} - I_{t-1} - R_t, 0). \quad (13)$$

We assume that the average cost of remanufacturing increases in the quantity of the products remanufactured. This assumption is reasonable since used products arrive in different quality levels, so an increase in q_r forces to firm to remanufacture cores of decreasing quality levels (Ferguson and Toktay, 2006). The case of remanufacturing cost being convex increasing in the quantity has been identified in several studies (e.g., Guide and Wassenhove, 2001). To model this phenomenon, we assume that the total cost to remanufacture q_r units is $c_r q_r^2$ such that an average cost of remanufacturing q_r units becomes $c_r q_r$. In our model, as q_r increases, the manufacturer has to remanufacture cores of decreasing quality levels and therefore, average cost of remanufacturing increases. One of the reasons why average cost increases is that we allow the manufacturer to buy used products from third-party core supplier, so these products may be of much lower quality than those returned by lessee at

the end of the lease term. The other reason is the existence of lease durations of different length. If products are used for a longer time, they depreciate more and hence residual value decreases. When q_r increases, the manufacturer has to use these low-quality products to remanufacture, which causes an increase in remanufacturing cost.

The cost of acquiring a used product from the third-party core supplier is given by c'_r . All returned and purchased remanufacturable products are remanufactured at an average cost of $c_r q_r$. In this setting, we assume that the unit cost of manufacturing, c_n , the unit cost of remanufacturing, $c_r q_r$, and the unit purchasing cost of used products, c'_r , are constant over the life cycle of the product. Finally, we allow the manufacturer to carry inventory of used products, but we do not consider associated holding costs to keep the focus on returns from lease agreements.

3.2 Model

We develop a discrete-time, multi-period, discounted profit optimization model. We assume that the shortest leasing duration is 12 months (one year) and lease agreements can differ from one to L periods, after which the product is returned to the manufacturer.

The model consists of continuous variables $\mathbf{p}_t = (p_{n,t}, p_{r,t})$ and $\mathbf{q}_t = (q_{n,t}, q_{r,t})$. Product prices are allowed to be time-dependent. The manufacturer chooses $\mathbf{p}_t = (p_{n,t}, p_{r,t})$ in period $t \geq 0$. Let β denote the discount factor over this time period. We omit t in our revenue and profit functions for the sake of notational convenience.

We also assume that customers are myopic, that is they do not take into account the future prices when making leasing or purchasing decisions. Hence, myopic (or non-strategic) customer behavior allows us to ignore any detrimental effect of future price cuts on current customer preferences.

The per-period revenue is given by

$$r_{\beta_m}(\mathbf{p}) = \left[\sum_{m=1}^L \frac{\beta_m (1 - \beta_m^{12m})}{1 - \beta_m} q_m(\mathbf{p}) p_m \right] + q_r(\mathbf{p}) p_r. \quad (14)$$

Then, the profit obtained in a period can be written as

$$\Pi_{\beta_m}(\mathbf{p}) = r_{\beta_m}(\mathbf{p}) - c_n q_n(\mathbf{p}) - c_r q_r^2(\mathbf{p}) - c'_r \max(0, \Delta_r). \quad (15)$$

Let $V_\beta(I)$ denote the optimal β -discounted multi-period profit of the manufacturer under the initial condition $I_0 = I$.

$$V_\beta(I) = \max \sum_{t=1}^T \beta^{t-1} \Pi_{\beta_m}(\mathbf{p}_t). \quad (16)$$

The optimal solution to this maximization problem is the multi-period price vector \mathbf{p}_t^* for new and remanufactured products.

4 Solution Procedure

Since the resulting problem is a nonlinear constrained optimization problem (NLP), the optimal solution cannot be obtained easily. Moreover, due to the complex structure of the problem, we are not able to analytically determine the unique optimal prices. In general, when the objective function is nonlinear and non-differentiable, or it is not convenient to use the information obtained from differentiation, some direct search methods are preferred. One of them is the Nelder-Mead Simplex Search Method (Nelder and Mead, 1965). Since the simplex search is originally developed to solve unconstrained problems, we modify it in order to handle price constraint. We give below the steps of the simplex search algorithm.

1. Construction of the initial simplex: Choose points $\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^{n+1}$ to form a simplex. Choose a reflection coefficient $\alpha > 0$, an expansion coefficient $\gamma > 1$, a contraction coefficient $0 < \lambda < 1$, and a shrinkage coefficient $\chi > 0$. Go to Step 2.
2. Initialization: Let $\mathbf{p}^{\min}, \mathbf{p}^{\max} \in \{\mathbf{p}^1, \dots, \mathbf{p}^{n+1}\}$ such that $\Pi(\mathbf{p}^{\max}) = \max_{1 \leq h \leq n+1} \Pi(\mathbf{p}^h)$, $\Pi(\mathbf{p}^{\min}) = \min_{1 \leq h \leq n+1} \Pi(\mathbf{p}^h)$. Let $\bar{\mathbf{p}} = \frac{1}{n} \sum_{h=1: \mathbf{p}^h \neq \mathbf{p}^{\min}}^{n+1} \mathbf{p}^h$. Go to Step 3.
3. Reflection: Let $\mathbf{p}^r = \bar{\mathbf{p}} + \alpha(\bar{\mathbf{p}} - \mathbf{p}^{\min})$.

If $\Pi(\mathbf{p}^r) \geq \Pi(\mathbf{p}^{\max})$, go to Step 4.

If $\Pi(\mathbf{p}^r) < \Pi(\mathbf{p}^{\max})$, but $\Pi(\mathbf{p}^r) \geq \min_{h:\mathbf{p}^h \neq \mathbf{p}^{\min}} \Pi(\mathbf{p}^h)$, then replace \mathbf{p}^{\min} by \mathbf{p}^r to form a new set of $n + 1$ points and go to Step 6.

4. Expansion: Let $\mathbf{p}^e = \bar{\mathbf{p}} + \gamma(\mathbf{p}^r - \bar{\mathbf{p}})$.

Replace \mathbf{p}^{\min} by \mathbf{p}^e if $\Pi(\mathbf{p}^r) < \Pi(\mathbf{p}^e)$ and by \mathbf{p}^r if $\Pi(\mathbf{p}^r) \geq \Pi(\mathbf{p}^e)$ to yield a new set of $n + 1$ points and go to Step 6.

5. Contraction: Let $\mathbf{p}^c = \bar{\mathbf{p}} + \lambda(\hat{\mathbf{p}}^{\min} - \bar{\mathbf{p}})$,

where $\hat{\mathbf{p}}^{\min}$ is defined as $\Pi(\hat{\mathbf{p}}^{\min}) = \max \{ \Pi(\mathbf{p}^{\min}), \Pi(\mathbf{p}^r) \}$. If $\Pi(\mathbf{p}^c) \geq \Pi(\hat{\mathbf{p}}^{\min})$, replace \mathbf{p}^{\min} with \mathbf{p}^c . If $\Pi(\mathbf{p}^c) < \Pi(\hat{\mathbf{p}}^{\min})$, replace \mathbf{p}^h with $\mathbf{p}^h + \chi(\mathbf{p}^{\max} - \mathbf{p}^h)$ for $h = 1, \dots, n + 1$. Go to Step 6.

6. Termination: If $\left\{ \frac{1}{n+1} \sum_{h=0}^n [\Pi(\mathbf{p}^h) - \Pi(\bar{\mathbf{p}})]^2 \right\}^{1/2} < \varepsilon$, then stop and set $\mathbf{p}^{best} \leftarrow \mathbf{p}^{\max}$, else go to Step 2.

Simplex search uses a polyhedron with $n + 1$ vertices for a problem with n variables to define the current simplex. Each vertex is represented by an n -dimensional vector. New candidate vectors are generated by reflections of some of the vectors and contractions around the vectors which correspond to a higher objective function value. The decision for the candidate vectors are made according to their objective value. By following this procedure, the simplex expands and contracts during the solution step and finally contracts to a single vector, which is a local optimum. We modify the simplex search so that after any update of the vertices of the simplex, the constraints are not violated. Therefore, the feasibility is preserved throughout the search.

Construction of the initial simplex is done by defining a price vector \mathbf{p}^h for each vertex of the simplex. Since we have two variables in our problem as p_n and p_r , we have to obtain $2T + 1$ points $\mathbf{p}^1, \mathbf{p}^2, \dots, \mathbf{p}^{2T+1}$ to form a simplex. For instance, if we solve a 5-period problem, we have 10 variables, therefore we will use 11 vertices to form the initial simplex. Moreover, each point consists of $p_{n,t}$ and $p_{r,t}$ for $t = 1, \dots, T$ where the first T components of the price

vector is constructed by $p_{n,t}$'s, and the rest of the vector by $p_{r,t}$'s. Therefore, we have $2T$ components at each vertex.

The first price vector is selected by generating prices arbitrarily in interval $[0, 1]$ considering also the price constraint which requires that $p_{r,t} \leq \delta p_{n,t}$ for $t = 1, \dots, T$. The rest of the simplex is constructed by using the suggestion in Bazaraa et al. (1993) with some modifications to satisfy the price constraint for the rest of the vertices as follows:

$$\mathbf{p}^{h+1} = \mathbf{p}^h + \mathbf{d}^h \quad h = 1, \dots, n. \quad (17)$$

where n denotes the number of variables. Here \mathbf{d}_h is a vector with h th component is equal to a and all other components equal to b with

$$a = \frac{s}{n\sqrt{2}} \left(\sqrt{n+1} + n - 1 \right), \quad (18)$$

$$b = \frac{s}{n\sqrt{2}} \left(\sqrt{n+1} - 1 \right), \quad (19)$$

where s is a scalar. Although we have $2T$ variables in our model, we set n to T while constructing components of \mathbf{d}_h since we need only T components for the first part of each vertex.

After we obtain the first part of each vertex, we construct the second part by multiplying each $p_{n,t}$ with a number generated randomly in interval $[0, \delta]$ to satisfy the price constraint. To avoid the risk of being trapped into a local optimum, we restart the simplex run 100 times with different step sizes, s . The values of $s = \{0.4, 0.5, 0.6, 0.7, 0.8\}$ works well. Here, we use $s = 0.4$ for initial 20 runs, and $s = 0.5$ for the following 20 runs, and etc. We select the best solution among the results of these 100 runs. The frequency of the local optima obtained during the search can be observed from standard deviation of the solution.

The experiments can be grouped in two main categories. In the first group, we solve single-period problems where the duration of leasing is one period. We perform sensitivity analysis with respect to the following parameters: consumers' relative willingness-to-pay for a

remanufactured product, relative willingness-to-pay for leasing a new product, deterioration of the product in age, initial inventory level, and cost of supplying used products from the third-party core supplier. In the second group, we solve multi-period problems in order to analyze the effects of returns of previously leased products on the manufacturer’s decision and profit. The effect of problem parameters mentioned above is also presented.

The simplex algorithm is coded in Visual C++ 6.0 environment and the experiments are run on a Pentium M, 1.7 MHz machine with 512 MB of RAM. As the run times are considerably short, no result is given for the CPU times.

5 Experimental Results

5.1 Single Period Problem

Single period optimization gives us general information about the problem parameters which affect the manufacturer’s decision. Since we design our experiments with single-period leasing option, the potential market consists of only one segment of customers who desire to lease a new product for one period, or buy a remanufactured product, or nothing, thus $\alpha_1 = 1$. Unless otherwise stated, parameter values shown in Table 4 have been fixed for the analysis.

Table 4: Fixed parameter values common for all experiments

c_n	c_r	i
0.1	0.05	0.08

5.1.1 The effect of the relative willingness-to-pay for remanufactured products

In this subsection, we keep all parameters fixed except δ to analyze its effect of the solution. Some parameter values are already given in Table 4, and d_1 , l_1 , c'_r , and I_0 are given in Table 5.

These values imply that a product loses 10 percent of its value in the first year ($d_1 = 0.1$), and a customer’s reservation price for leasing a new product for one period is half of his

Table 5: Fixed parameter values to analyze the effect of δ

d_1	l_1	c'_r	I_0
0.1	0.5	0.08	0

reservation price for buying a new product ($l_1 = 0.5$). If the inventory of used products is not adequate to meet the demand for remanufactured products, the manufacturer purchases the necessary cores from the third-party supplier with cost $c'_r = 0.08$. We assume that there exists no inventory at the beginning of time horizon, i.e., $I_0 = 0$. We conduct our experiments in this section for different δ values ranging from 0.2 to 0.9.

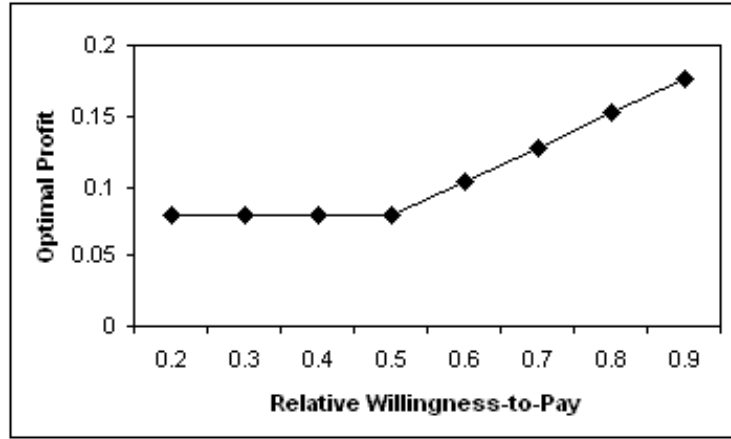


Figure 1: The effect of δ on the profit

Figure 1 illustrates the change in the optimal profit as δ increases. If δ is high, customers consider the new and remanufactured products almost identical, and are willing to pay almost the same amount for either product, which results in an increase in the profit. When $l_1 > \delta$, the increase δ does not affect the profit since there is no demand for remanufactured products. When $l_1 \leq \delta$, the profit of the manufacturer increases at an increasing rate due to the increase of q_r and p_r . Optimal prices and demands are given in Table 6.

As long as $\delta < l_1$, there is no demand for remanufactured products as seen from Figure 2 since customers value remanufactured products less than new products which prevents the manufacturer from having profit from the remanufactured product. Thus, for low δ values, it is more profitable to encourage consumers to lease new products than to buy remanufactured products. When $\delta = l_1$, there is no demand for new products and most of

Table 6: Optimal prices and demands for different δ values

δ	p_n	p_r	q_n	q_r	q_0
0.2	1.7793	0.1468	0.4000	0.0000	0.6000
0.3	1.7793	0.3159	0.4000	0.0000	0.6000
0.4	1.7793	0.6624	0.4000	0.0000	0.6000
0.5	2.1676	0.3091	0.0000	0.3818	0.6182
0.6	1.9000	0.3600	0.0000	0.4000	0.6000
0.7	2.3063	0.4107	0.0000	0.4133	0.5867
0.8	1.8197	0.4612	0.0000	0.4235	0.5765
0.9	1.9095	0.5116	0.0000	0.4316	0.5684

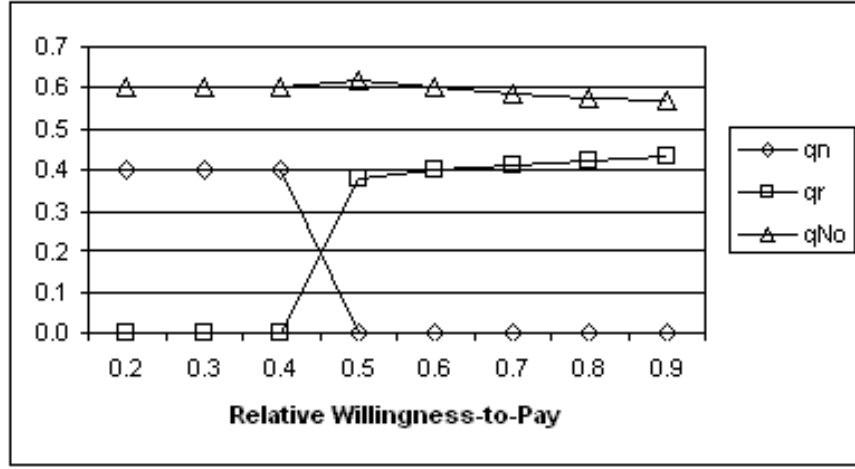


Figure 2: The effect of δ on the demand

the consumers shift towards the remanufactured products. Thus, at this point, the volume of consumers who prefer nothing increases. From this point on, it is seen that as δ increases, the amount of remanufactured products increases and the volume of consumers who prefer nothing decreases.

If $\delta < l_1$, the price of new products does not change and price of remanufactured products is charged higher with respect to δ in order to create demand for only new products. However, if $\delta \geq l_1$, the price of remanufactured products increases to take advantage of the increased willingness-to-pay. In this case we observe that the profit of the manufacturer increases due to the increase of q_r and p_r as illustrated in Figure 3.

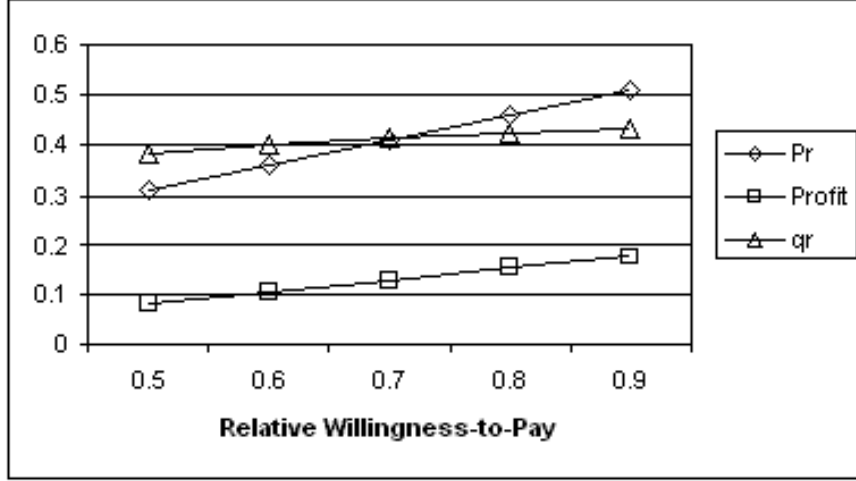


Figure 3: The effect of δ on q_r , p_r and profit if $\delta \geq l_1$

5.1.2 The effect of the relative willingness-to-pay for leased products

We keep all parameters fixed except l_1 to analyze its effect. We can only observe the effect of l_1 in those cases when there is demand for new products. Therefore, we set δ to 0.2. The fixed parameter values are given in Table 7.

Table 7: Fixed parameter values to analyze the effect of l_1

d_1	δ	c'_r	I_0
0.1	0.2	0.08	0

When the effect of changes in l_1 is analyzed for $\delta = 0.2$, similar results are found to the cases while analyzing the effect of δ with $l_1 = 0.5$. This is because each consumer's willingness-to-pay for leasing a new product for one period and for buying a remanufactured product are fractions l_1 and δ , respectively, of their willingness-to-pay for buying a new product. Similar to the effect of δ , the price of the new product increases as l_1 increases to take advantage of increased willingness-to-pay for leasing. Therefore, the higher the l_1 value, the higher is the profit as illustrated in Figure 4. However, if $l_1 \leq \delta$, the increase in l_1 does not affect the profit since there is no demand for new products. Optimal prices and demands are given in Table 8.

For $l_1 \leq \delta$, there is no demand for the new product since consumers have more utility

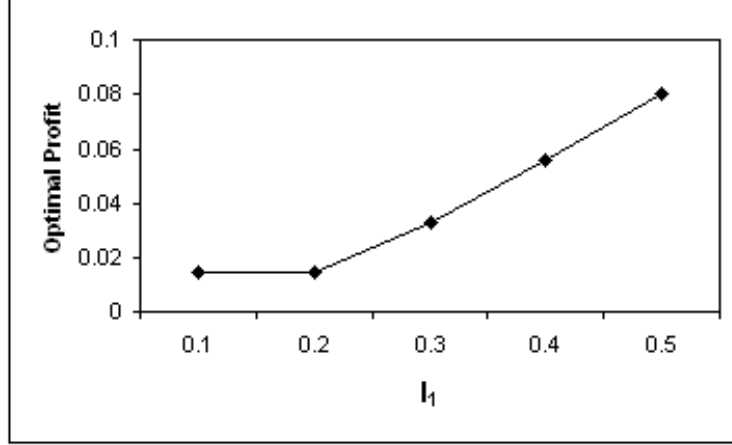


Figure 4: The effect of l_1 on the profit

Table 8: Optimal prices and demands for different l_1 values

l_1	p_n	p_r	q_n	q_r	q_0
0.1	1.5200	0.1520	0.0000	0.2400	0.7600
0.2	1.2400	0.1520	0.0000	0.2400	0.7600
0.3	1.1862	0.2372	0.3333	0.0000	0.6667
0.4	1.4828	0.2040	0.3750	0.0000	0.6250
0.5	1.7793	0.1926	0.4000	0.0000	0.6000

from buying remanufactured product than leasing a new product due to the low perception on leasing. However, when $l_1 > \delta$, namely a customer's valuation for leasing a new product for one period is at 30 per cent of his valuation for buying a new product and thus greater than his valuation for buying a remanufactured product, new products are preferred by customers. From this point on, it is seen that as l_1 increases, the amount of new products increases and the volume of consumers who prefer nothing decreases. Figure 5 exhibits the trends on demands as l_1 increases.

It is important to note that if both l_1 and δ decrease, the volume of customers who prefer nothing increases which results in a decrease in the manufacturer's profit since both demands and prices decrease. We plot the optimal profit versus l_1 for different values of δ (ranging from 0.2 to 0.5) in Figure 6.

When the joint effect of l_1 and δ is analyzed, it is seen that the profit increases in l_1 only if l_1 is greater than δ . This is because the volume of the new products increases in l_1 only if

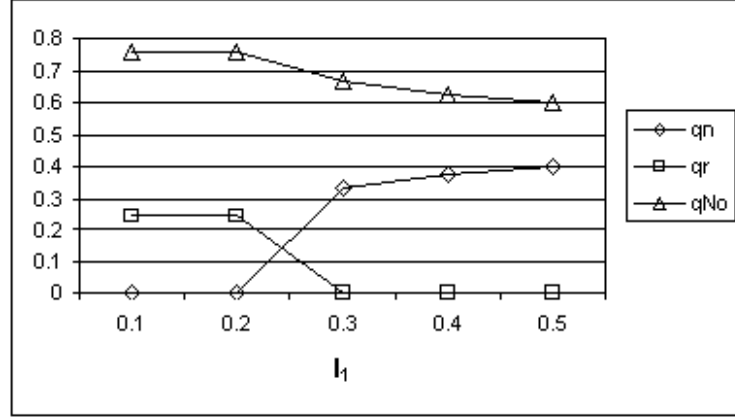


Figure 5: The effect of l_1 on the demand

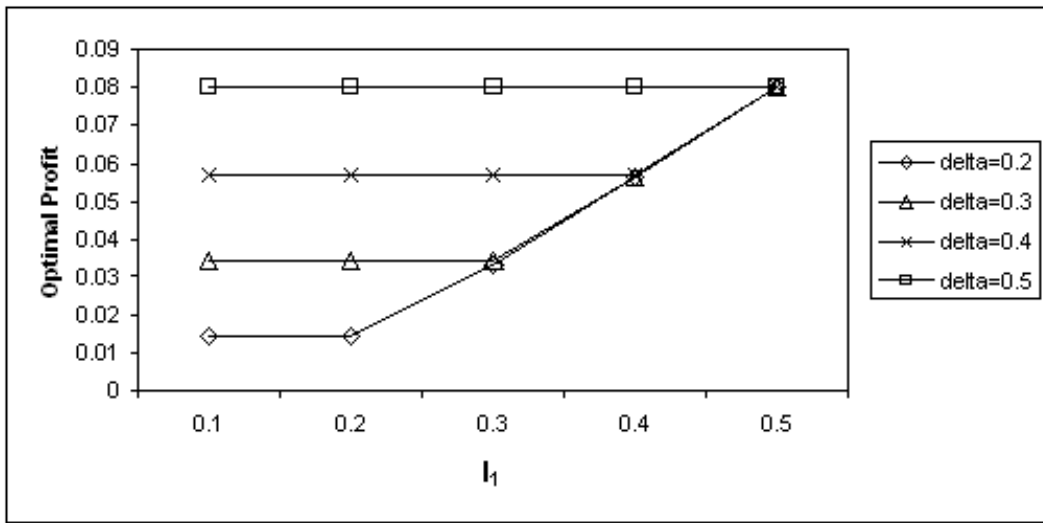


Figure 6: The effect of l_1 on the profit for different δ values

$l_1 > \delta$. If l_1 is less than or equal to δ , consumers do not change their preferences because of the fact that they derive more utility from buying a remanufactured product. For instance, in Figure 6, when $\delta = 0.5$, the profit does not change since l_1 values are less than or equal to δ , and consumers prefer remanufactured products.

5.1.3 The effect of the product deterioration

We analyze the effect of the product deterioration by changing the values assigned to parameter d_1 . Using the fixed parameter values given in Table 9, we vary d_1 between 0.1 and 0.6 with increments of 0.1.

Note that an increase in d_1 results in increasing lease payments p_1 . This can be seen in

Table 9: Fixed parameter values to analyze the effect of d_1

l_1	δ	c'_r	I_0
0.5	0.2	0.08	0

Equation (1), which is given as

$$p_m = p_n K_m \quad m = 1, \dots, L, \quad (20)$$

with

$$K_m = \left(\frac{d_m}{12m} + (2 - d_m) MF \right). \quad (21)$$

As Table 10 illustrates, higher d_1 values yield increased K_1 values.

Table 10: K_1 values for different d_1 values

d_1	0.1	0.2	0.3	0.4	0.5	0.6
K_1	0.014667	0.022667	0.030667	0.038667	0.046667	0.054667

Table 11 illustrates the optimal prices and demands for different d_1 values. As d_1 increases, p_n decreases. However, since K_1 increases with d_1 , p_1 values do not change for d_1 values between 0.1 and 0.4. Furthermore, as p_n decreases, p_r also decreases. Since δ is low, namely consumers see remanufactured products almost worthless, q_r does not change. Therefore, it is obvious that there is a threshold for depreciation rate such that for values less than this threshold, there is no change in the demand even though d_1 increases. On the other hand, when the threshold is exceeded, q_n decreases and q_r increases as d_1 increases. This is because the decrease in p_n forces p_r to decrease due to the constraint $p_r \leq \delta p_n$. Although consumers with high willingness-to-pay for leasing a new product still prefer leasing, consumers with low willingness-to-pay shift towards the remanufactured product due to the decrease in the price of the remanufactured product. This decrease of prices affects the profit negatively. In Figure 7, we exhibit the effect of d_1 on the prices and demands.

Table 11: Optimal prices and demands for different d_1 values when $l_1 = 0.5$, $\delta = 0.2$

d_1	p_n	p_r	q_n	q_r	q_0	$PV(p_1)$	p_1
0.1	1.7793	0.2411	0.4000	0.0000	0.6000	0.3000	0.0261
0.2	1.1513	0.2231	0.4000	0.0000	0.6000	0.3000	0.0261
0.3	0.8510	0.1702	0.4000	0.0000	0.6000	0.3000	0.0261
0.4	0.6749	0.1350	0.4000	0.0000	0.6000	0.3000	0.0261
0.5	0.5526	0.1105	0.3803	0.0672	0.5526	0.2964	0.0258
0.6	0.4488	0.0898	0.3590	0.1922	0.4488	0.2821	0.0245

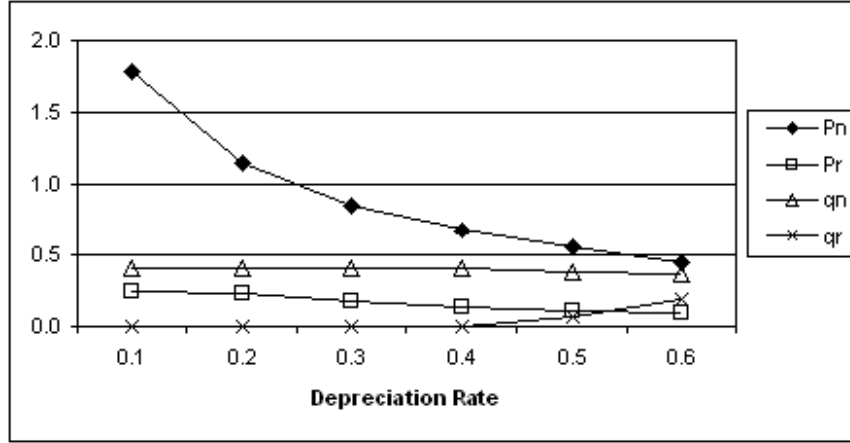


Figure 7: The effect of d_1 on prices and demands when $l_1 = 0.5$, $\delta = 0.2$

5.1.4 The effect of the unit cost of cores acquired from third-party core supplier

Table 12 gives the fixed parameter values used in the analysis of the effect of c'_r .

Table 12: Fixed parameter values to analyze the effect of c'_r

l_1	d_1	δ	I_0
0.5	0.1	0.2	0

$c'_r = 0$ corresponds to the case when there is no constraint with respect to remanufactured products' sales, namely $I_0 = 1$. Thus, the average cost of remanufacturing used cores is given only by $c_r q_r$. On the other hand, an increase in c'_r makes a remanufactured product less attractive with respect to a new product. Thus, the volume of remanufactured products sold decreases as c'_r increases and optimal pricing becomes such that the demand for remanufactured products decreases so as to reduce the detrimental effect of the increase in c'_r on the profit. The increase in the price of the remanufactured product in such a way

that demand for remanufactured products decreases and demand for new products increases makes the profit decrease as illustrated in Figure 8. This is due to the fact that the profit gain obtained from the increase in the amount of leased new products is less than the profit loss due to the decrease in the amount of remanufactured products sold.

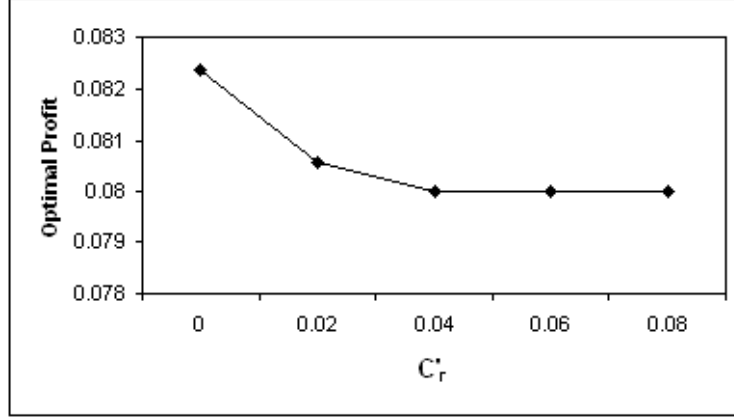


Figure 8: The effect of c'_r on the profit

When the demand for remanufactured products becomes zero due to the high c'_r , a further increase in this parameter does not affect decisions on the demand and profit as given in Table 13.

Table 13: Optimal prices and demands for different c'_r values when $l_1 = 0.5$, $\delta = 0.2$

c'_r	p_n	p_r	q_n	q_r	q_0
0.00	1.7793	0.1059	0.3529	0.1176	0.5294
0.02	1.7793	0.1129	0.3765	0.0588	0.5647
0.04	1.7793	0.2567	0.4000	0.0000	0.6000
0.06	1.7793	0.2038	0.4000	0.0000	0.6000
0.08	1.7793	0.1705	0.4000	0.0000	0.6000

When the effect of c'_r is analyzed for different δ values, similar results are obtained. Figure 10 presents the behavior of the optimal profit as c'_r increases for different δ values.

5.2 Multi-Period Problem

Due to the interdependence of new and remanufactured products, a decrease in demand for new products results in a decrease in the availability of used products that are remanufac-

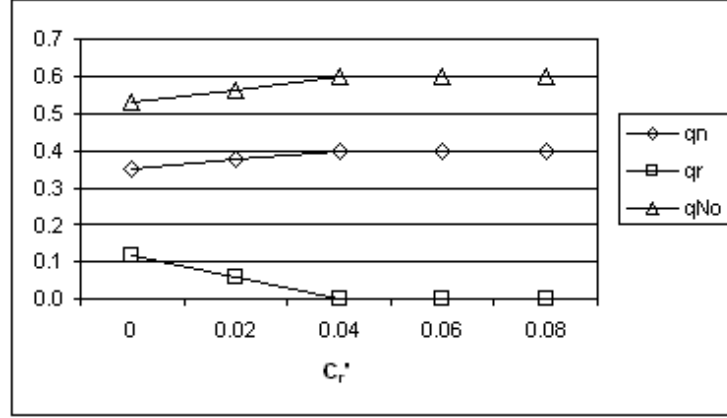


Figure 9: The effect of c'_r on the demands when $l_1 = 0.5$, $\delta = 0.2$

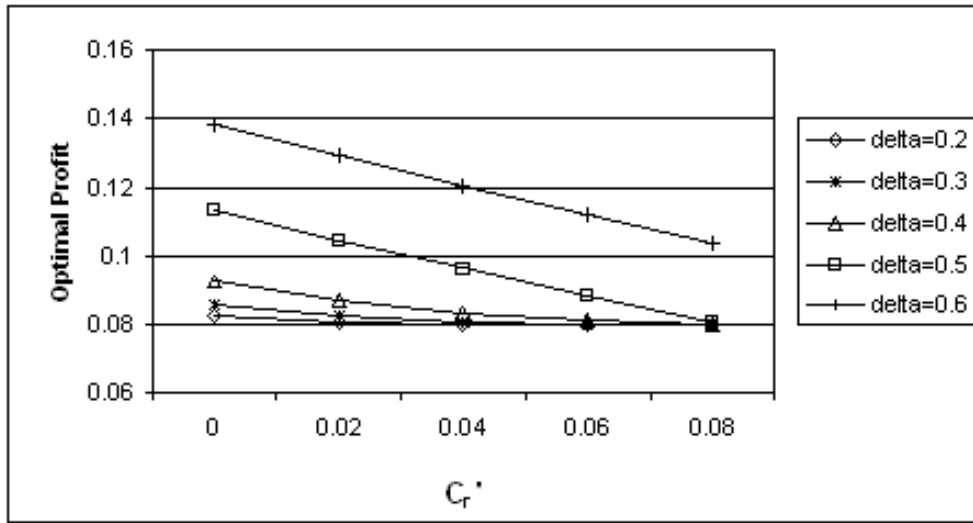


Figure 10: The effect of c'_r on the profit for different δ values

tured. In the previous section, we could not investigate the impact of past lease decisions on the future decisions of the firm. In the multi-period setting, it is possible to investigate the interdependence of new and remanufactured products since all previously leased products have to return at the end of the lease term, which are further used for remanufacturing.

In this section, we investigate the implications of this dependency on the pricing strategy. For instance, the manufacturer may choose to produce some new products only for the future value that they generate through their sale as remanufactured products, although these products are sold at a loss currently.

The analysis is considered up to four periods, and in each period consumers decide

whether to lease a new product or to buy a remanufactured product based on the value of consumer surplus in that period. In the multi-period setting, we only investigate the effect of the relative willingness-to-pay for remanufactured products. Therefore, we keep all parameters fixed except δ . Parameter values that are fixed are given in Tables 4 and 5.

For the one-period leasing option, new products return at the end of one year. As mentioned before, the supply of used products that become available at the beginning of time t is R_t . The volume of used products that remain in stock from returns in previous periods at the beginning of period t is I_{t-1} . $I_{t-1} + R_t$ gives us the available inventory at the beginning of the current period.

If there is inventory of used products when $\delta < l_1$, demand for remanufactured products may be positive such that q_r does not exceed the available inventory. If the maximum q_r value \bar{q}_r for a given δ is greater than I_0 , then, as I_0 increases, q_r increases as well. Thus, in a multi-period problem, if returns are adequate for remanufacturing, q_r becomes positive in the second period as illustrated in Table 14 for two-period problem when $l_1 = 0.5$ and $\delta = 0.2$.

Table 14: Optimal prices and demands for 2 periods, $l_1 = 0.5$, $\delta = 0.2$

Periods	p_n	p_r	q_n	q_r	q_0	I_t	$PV(p_1)$	p_1
1	1.7793	0.2982	0.4000	0.0000	0.6000	0.0000	0.3000	0.0261
2	1.7793	0.1059	0.3529	0.1176	0.5294	0.2824	0.3000	0.0261

In the single period model, when $l_1 = \delta$, there is demand only for remanufactured products, but in the multi-period setting this is not the case due to the threat of supply of remanufacturable products in the next period. Thus, in the first period, demand for new products exists, and the manufacturer starts the second period with the available $q_{n,1}$ cores that were leased in the previous period. If the problem were solved period by period without considering the interdependence of new and remanufactured products, selling more remanufactured products would seem to be more profitable without considering the next period. However, if the manufacturer chooses to produce some new products only for the future value that they generate through their sale as remanufactured products, he makes more profit. In

this framework, the optimal pricing is such that the demand for new products is positive in the first period, while the demand for remanufactured products is positive in the second period. This comparison is given in Table 15.

Table 15: Profit comparison for two-period problem when $l_1 = 0.5$, $\delta = 0.5$

		Demands			
		Simultaneous Decision		Period by Period	
		$q_{n,t}$	$q_{r,t}$	$q_{n,t}$	$q_{r,t}$
1. Period	0.4275	0	0	0.3818	
2. Period	0	0.4275	0	0.3818	
Profit	0.184468		0.154424		

Table 16 includes the optimal prices and demands if $l_1 = 0.5$ and $\delta = 0.6$. Recall that in the single period problem if $l_1 < \delta$, there is demand only for remanufactured products. However, in the multi-period setting, the demand for both new and remanufactured products is positive in the first period so as to decrease the amount supplied from the third-party core supplier in the next period.

Table 16: Optimal prices and demands - 2 periods, $l_1 = 0.5$, $\delta = 0.6$

Periods	p_n	p_r	q_n	q_r	q_0	I_t	$PV(p_1)$	p_1
1	1.5596	0.3477	0.3210	0.1531	0.5259	0.0000	0.2630	0.0229
2	1.7794	0.3600	0.0000	0.4000	0.6000	0.0000	0.3000	0.0261

In Figures from 11 to 16, we exhibit the effect of δ on the demand for new and remanufactured products in the first and last period of two-period, three-period, and four-period problems, respectively. In the first period, when $\delta < l_1$ and $I_0 = 0$, q_r does not increase as δ increases because of the fact that q_r does not exceed I_0 due to the extra cost c'_r . We also see from Figure 11 that there is no demand for remanufactured products since customers value remanufactured products less than new products. This causes the manufacturer not to charge high p_r values and, therefore not to obtain profit by selling the remanufactured product. To create demand for new products in the first period is beneficial from the perspective of both manufacturer's profit and the supply of used products in the next period. When $l_1 = \delta$, consumers are indifferent between leasing a new product and buying a re-

manufactured product. Therefore, they prefer the one which has less price. In this case, manufacturer charges higher prices for remanufactured product to shift consumers towards new products in the first period considering the supply of used products in the future. q_r is positive for the first time when $\delta > l_1$ and, from this point on, q_n decreases (q_r increases) as δ increases.

The manufacturer starts the last period with the opportunity to remanufacture used products that were leased in previous periods and become available at the beginning of the last period. q_r increases (q_n decreases) in δ such that q_r does not exceed the volume of returns when $\delta < l_1$. For $\delta \geq l_1$, q_r may exceed the stock of used products since there are consumers who are willing to pay high prices for remanufactured products which creates profit although the shortage in used products is supplied from the third-party core supplier with an extra cost c'_r .

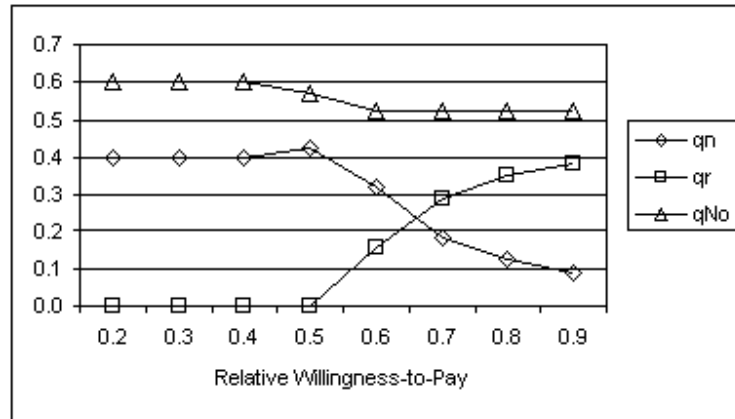


Figure 11: Effect of δ on the demand in the first period of a 2-period problem

Until now, we have presented the effect of changes in δ on demands for the first and last period of the multi-period problems for different T values. In periods stated between first and last periods of the time horizon, q_r increases in δ when $\delta < l_1$ and $\delta > l_1$. But when $\delta = l_1$, this trend changes such that q_r increases too much due to the returns. However, when $\delta > l_1$, since demand for remanufactured products exists in each period, the volume of new products decreases and also causes the volume of the remanufactured products to decrease in future periods. From this point on, q_r continues to increase with respect to δ . Figure 17 illustrates this phenomenon.

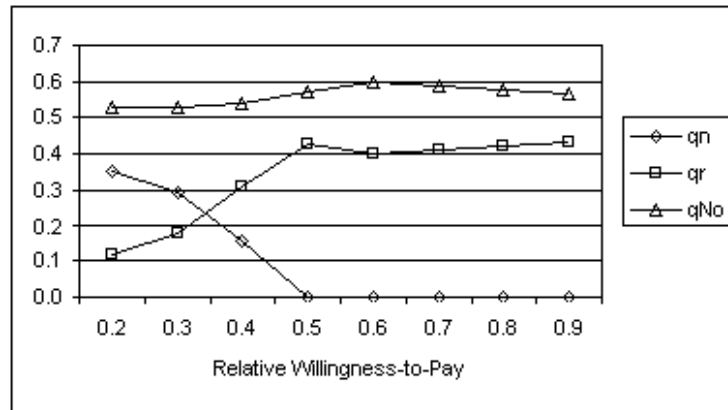


Figure 12: Effect of δ on the demand in the last period of a 2-period problem

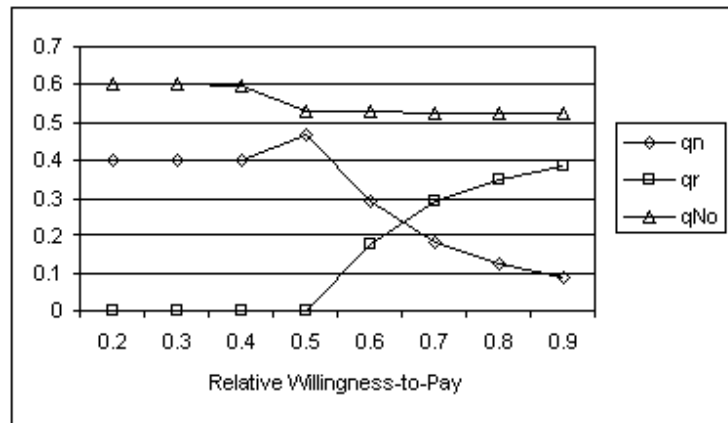


Figure 13: Effect of δ on the demand in the first period of a 3-period problem

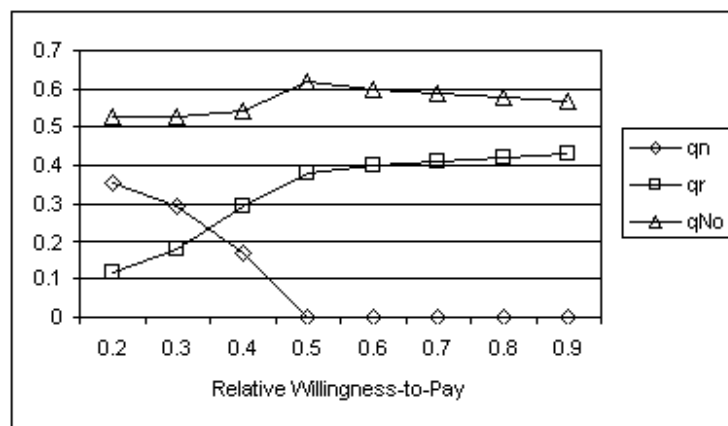


Figure 14: Effect of δ on the demand in the last period of a 3-period problem

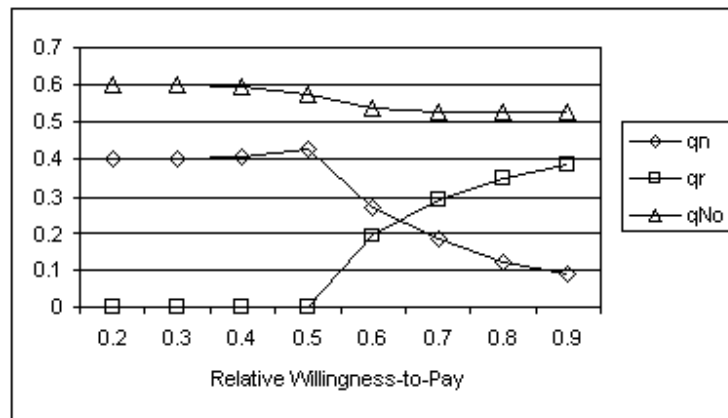


Figure 15: Effect of δ on the demand in the first period of a 4-period problem

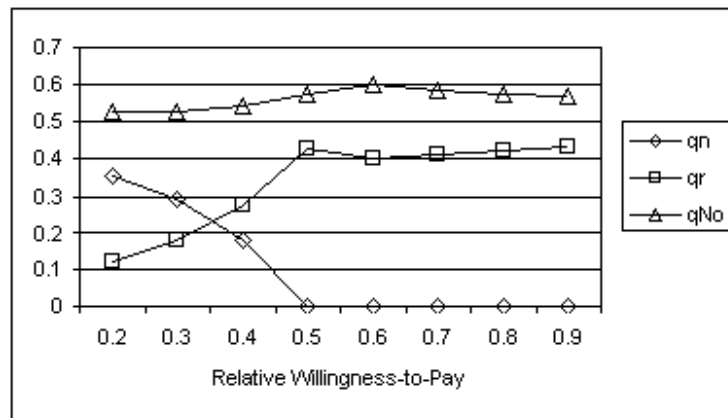


Figure 16: Effect of δ on the demand in the last period of a 4-period problem

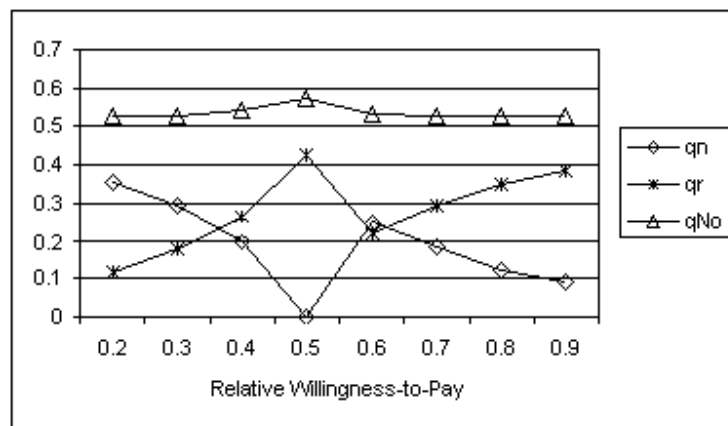


Figure 17: Effect of δ on the demand in the second period of a 4-period problem

As δ increases, p_r also increases to take advantage of increased willingness-to-pay which makes the profit increase. In Figure 18, we present the optimal profit against the relative willingness-to-pay δ for remanufactured products for different time horizons up to five periods. Note that profit curves are increasing.

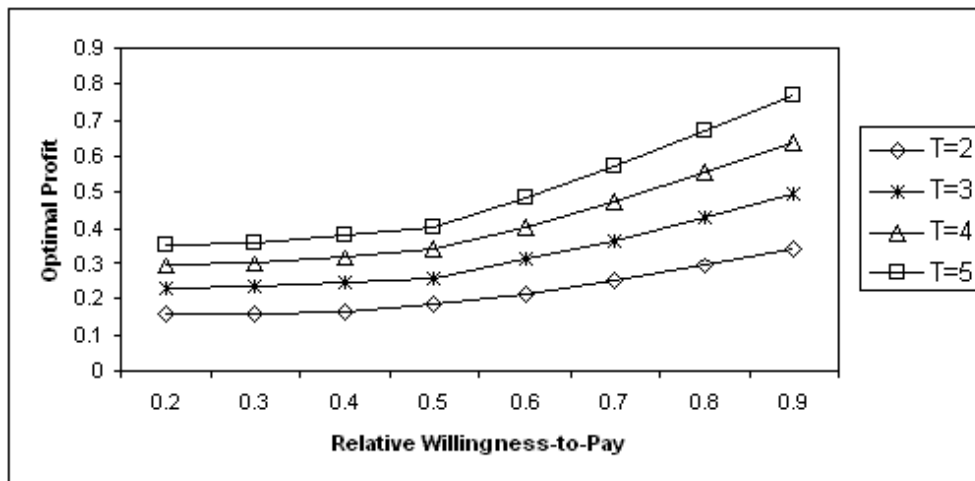


Figure 18: Change in the optimal profit as a function of δ

6 Conclusion

The aim of this paper is to determine the optimal pricing strategy in a multi-period setting for a profit-maximizing firm leasing new, durable, and remanufacturable products as well as selling remanufactured products to a customer base that has a lower willingness-to-pay for the remanufactured product. If available used products are not enough to meet the demand for remanufactured products, the manufacturer acquires the remaining from the third-party core supplier with an extra cost. We formulate a profit maximizing model using the notion of consumer surplus. The resulting problem is solved by a variant of Nelder-Mead simplex search method which can also handle constraints.

Experimental are performed for single-period problems with respect to the different problem parameters such as relative willingness-to-pay for remanufactured products, relative willingness-to-pay for leasing new products, depreciation rate of the product over lease period, and cost c'_r of supplying used products from the third-party core supplier.

In the multi-period setting, we investigate the interdependence of new and remanufactured products such that a decrease in the demand for new products in prior periods results in a decrease in the availability of used products. This is because decisions in a given period explicitly depend on decisions in previous periods in such a scenario where the manufacturer acquires used cores through leasing new products. In this framework, we find that the manufacturer may choose to produce some new products only for the future value that they generate through their sales as remanufactured products. If there is no available stock of used cores at the beginning of the time horizon, and consumers have higher willingness-to-pay for leasing a new product than buying a remanufactured product, the manufacturer does not produce remanufactured products in the first period due to the extra cost of supplying used items from the third-party core supplier. This is because of the fact that remanufacturing results in a significant drop in the profit in the scenario where consumers are willing to pay low prices for remanufactured products. However, in the subsequent periods, the manufacturer starts the period with the opportunity to recover used cores that become available at the beginning of the period and remain in stock from returns in previous periods. Therefore, it favors remanufacturing in the rest of the time horizon.

As a future work, it can be interesting to extend this model by considering variable costs which depend on the technological development. We assume that the costs of manufacturing and remanufacturing are constant over the time horizon, but in practice, they can be reduced by a new technology. Moreover, we allow the manufacturer to carry inventory of used products, but we do not consider associated holding costs. Therefore, it can be an extension of our model. Finally, we assume that the manufacturer holds a monopoly in the markets for new and remanufactured products. To capture the impact of competition in the remanufactured product market, the model can be extended by considering an industry in which the manufacturer holds a monopoly in the new product market and independent remanufacturers compete on the remanufactured product market. Since we assume new products are only leased, and return at the end of the lease period, the selling option of new products can be added to our scenario by considering that they can be collected by

independent remanufacturers.

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