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To estimate the stage completion time and the internal due date of job with dynamic arrival for a two-stage flexible flow shop

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Abstract: Considering the two-stage flexible flow shop with dynamic job arrival, each stage contains identical parallel machines, and the second stage receives jobs with stochastic arriving time from first stage. To estimate the job completion time at each stage is useful to enhance the customer service or production performances. The completion time of jobs at each stage can be determined with the sum of the stage arriving time, the expected stage waiting time, and the expected stage service time of jobs. The expected stage waiting time of job relates to the utilization rates of machines, which should be corrected due to the idleness of machine. The idle time depends on the number of jobs in the system at specific time. The birth-and-death process can be used to describe probabilistically how the number of jobs in the system changes as time increasing. Note the stage service time of jobs relates to the estimated setup time and the processing time of jobs at each stage. To analyze the accuracy on the estimated stage completion time of jobs, a computational test through simulation is conducted to verify the proposed probabilistic model.

Keywords: dynamic arrival of job, flexible flow shop, stage completion time, idle time, birth-and-death process

1. INTRODUCTION

The flexible flow shop with jobs arriving dynamically is a complicated scheduling problem, which has many real-world applications, especially in the TFT-LCD manufacturing or in the integrated circuit (IC) manufacturing industry. In general, the flexible flow shop consists of a set of more than two process stages, in which each stage is made up of identical parallel machines, and the next stage will receive jobs with stochastic arriving time from the previous stage. Jobs arrive at the system dynamically, which makes the arriving time of jobs at each stage and the completion time of jobs at each stage difficult to predict. Note that the final stage completion time can be applied to assign an internal due date for a job to respond the delivery date request from customer and to facilitate the shop floor control. Therefore, it is important to develop an efficient approach to estimate the job finished time at each stage to enhance the customer service and production performance.

In this paper, a probabilistic model is developed to estimate the stage completion time of jobs for the two-stage flexible flow shop, in which the inter-arrival time of jobs for different product types is assumed to be independently and exponentially distributed and the jobs are dispatched to parallel machines by FIFO rule. A sequence-dependent setup occurs for the process changing from one product type to another. The completion time of jobs at each stage can be determined with the summation of the stage arriving time of jobs, the expected stage waiting time of jobs, and the expected stage service time of jobs. The expected stage waiting time of job relates to the utilization rates of machines, which should be corrected due to the idleness of machine. The idle time depends on the number of jobs in the system at specific time. The birth-and-death process can be used to describe probabilistically how the number of jobs in the system changes as time increasing. The stage service time of jobs is defined by adding the estimated setup time and the processing time of jobs at each stage. The completion time of jobs at each stage relates to the arrival rates of jobs at each stage, therefore, the arrival rates of jobs at each stage must compute in advance. The arrival rates of jobs at incumbent stage are defined as the inverse of the mean of the difference of the completion time of jobs at previously stage.

The stage completion times of jobs are estimated in this paper, which has little been addressed by research works in the literature. In the past, related researches aimed at modeling the performance of dynamic production system, such as the mean flow time, the mean queue length, machine utilization or WIP. The performance measures can be estimated by queueing model (Karmarkar *ed al*. (1985a) and (Karmarkar 1985b)) Furthermore, the approximate models have been developed to estimate the performance measures under GI/G/m (Whitt (1993)). Vieira *et al*. (2000a) and Vieira *et al*. (2000b) presented the analytical models that can predict the performance like average flow time, machine utilization, and setup frequency. As regards the flexible flow shop, Co and Li (1989) presented a mean value analysis model to estimate the performance measures include the mean queue length, the mean flow time, and the expected throughput. Nieuwenhuyes and Vandaele (2006) presented the analytical expressions to estimate the average process time batch flow times for a two-stage stochastic manufacturing system.

To analyze the accuracy on the estimated stage completion time of jobs, calculated by the probabilistic model, a simulation model is built, in which jobs are dispatched according to the FIFO rule and the inter-arrival time of jobs was independently and exponentially distributed. The simulation runs are conducted for various parameters and the computational analysis is provided to demonstrate the effectiveness of the proposed probabilistic model. This paper is organized as follows: Section 2 develops the general probabilistic model to estimate the job completion time at each stage for the flexible flow shop. Section 3 presents the simulation results and the accuracy analysis of the probabilistic model. Section 4 gives the conclusions.

2. THE PROBABILISTIC MODEL FOR THE STAGE COMPLETION TIME

In this section, a probabilistic model for the stage completion time of jobs is developed for the two-stage flexible flow shop with jobs of different product types arriving dynamically, in which the inter-arrival time of jobs is exponentially distributed and the FIFO rule is applied to dispatch jobs. A sequence-dependent setup occurs with process changing from one product type to another. The general models of jobs for the second stage estimated completion time are developed by firstly defining the estimated completion time of jobs at the first stage and then secondly defining the estimated arrival rates.

2.1 The estimated completion time of jobs at the first stage

Let J be the number of product types and k_1 be the number of machines at the first stage. Let $N_{1,j}(t)$ be the number of arriving jobs of product type j by time t at the first stage, in which $\{N_{1,j}(t), t \geq 0\}$ assumes a Poisson process with arrival rate $\lambda_{1,j}$, $j = 1, 2, \ldots, J$. The total arrival rate at the first stage λ_1 is equal to the sum of $\lambda_{1,j}$ for all product types, $\sum_{j=1}^{J} \lambda_{1,j}$.

The flexible flow shop is made up of identical parallel machines at each stage. Since the jobs are dispatched to one of parallel machines at each stage, the expected completion time of the i^{th} job of type j at the first stage ($E(C_{1,ij})$) can be estimated with the sum of the expected arrival time of the i^{th} job of type j at the first stage ($E(Y_{1,[i]j})$), the expected waiting time in queue for the i^{th} job of type j at the first stage ($E(W_{1,ij})$), and the expected average service time for jobs of the product type j at the first stage ($E(ST_{1,ij})$). Therefore, $E(C_{1,ij})$ can be expressed as Equation (1) and the calculations of $E(Y_{1,[i]j})$, $E(W_{1,ij})$ and $E(ST_{1,ij})$ are presented below.

$$E\left(C_{1,ij}\right) = E\left(Y_{1,[i]j}\right) + E\left(W_{1,ij}\right) + E\left(ST_{1,ij}\right) \tag{1}$$

Since the inter-arrival time of job for the product type j at the first stage is independently and exponentially distributed with the parameter $\lambda_{1,j}$, $j = 1, 2, \ldots, J$, the arrival time of the product type j job in a horizon at the first stage should be uniformly distributed over $[L_1, U_1]$. Let $Y_{1,[i]j}$ be the arrival time of the i^{th} job of type j at the first stage under the condition that there are N_j product type j jobs arriving in the horizon $[L_1, U_1]$. The probability density function of $Y_{1,[i]j}$ is shown as Equation (2).

$$g_{Y_{1,[i]j}}\left(Y_{1,[i]j}\right)=\frac{N_j!}{(i-1)!\left(N_j-i\right)!}\left(\frac{y_{1,[i]j}}{U_1-L_1}\right)^{i-1}\frac{1}{U_1-L_1}\left(1-\frac{y_{1,[i]j}}{U_1-L_1}\right)^{N_j-i} \tag{2}$$

According to Equation (2), the expected value of $Y_{1,[i]j}$ can be calculated and is equal to $[i/(N_j+1)](U_1-L_1)$.

Consider identical parallel machines at the first stage where jobs with different product types arrive dynamically in the horizon $[L_1, U_1]$. The service time of job is equal to the sum of job's processing time and setup time, which is sequence-dependent. Therefore, the calculation of the service time of jobs can be divided into three situations. First, the service time of jobs would be zero with the condition no jobs arriving in the horizon $[L_1, U_1]$. Second, the service time of jobs would be equal to its processing time with the condition one job arriving in the horizon $[L_1, U_1]$ and this job requiring no setting up. Third, the service time of jobs would be equal to the sum of job's processing time and its setup time with the condition one job arriving in the horizon $[L_1, U_1]$ and this job requiring a setup.

Suppose that the inter-arrival time of product type j job at the first stage is independently and exponentially distributed with the parameter $\lambda_{1,j}$, then the probability of the i^{th} job of type j arriving at the system in the horizon $[L_1, U_1]$ can be calculated as Equation (3).

$$\Pr\left[L_1\le T_{1,ij}\le U_1\right]=\int_{L_1}^{U_1}\frac{\left(\lambda_{1,j}\right)^i}{\Gamma(i)}\left(t_{1,ij}\right)^{i-1}e^{\lambda_{1,j}t_{1,ij}}dt_{1,ij} \tag{3}$$

where $T_{1,ij}$ is the random variable that the time until the the i^{th} job of type j arrived at the system in the horizon $[L_1, U_1]$, and the probability density function of $T_{1,ij}$ is the gamma distribution with parameters $\lambda_{1,j}$ and i. Let $ST_{1,ij}$ be the random variable of the service time of the i^{th} job of type j at the first stage, $j=1,2,\ldots,J$. The probability mass function of $ST_{1,ij}$ can be calculated as Equation (4).

$$P\left(ST_{1,ij}=st_{1,ij}\right)=\begin{cases}\Pr\left[L_1\le T_{1,ij}\le U_1\right]\left(1-P_{1,FIFO}\right) & \text{, if } st_{1,ij}=pt_{1,j}\\ \Pr\left[L_1\le T_{1,ij}\le U_1\right]P_{1,FIFO}\dfrac{\lambda_{1,r}}{\lambda_1'} & \text{, if } st_{1,ij}=pt_{1,j}+s_{1,jr},\ r=1,2,\ldots,J,\ r\neq j\\ 1-\Pr\left[L_1\le T_{1,ij}\le U_1\right] & \text{, if } st_{1,ij}=0\end{cases} \quad (4)$$

where $pt_{1,j}$ represents the processing time of product type j at the first stage, $s_{1,jr}$ represents the setup time of product type j job at the first stage, in which the previous job belongs to product type r, $P_{1,FIFO}$ represents the probability that setup is required for the job of product type j at the first stage, where jobs are dispatched by the FIFO rule, $\lambda_{1,r}/\lambda_1'$ represents the probability that the previous job processed on the machine at the first stage belonging to product type r with $\lambda_1'=\sum_{r=1,r\neq j}^{J}\lambda_{1,r}$. According to the probability mass function of ST_{ij}, the expected service time of the i^{th} job of type j at the first stage $E(ST_{1,ij})$ can be calculated by Equation (5).

$$E\left(ST_{1,ij}\right)=\Pr\left[L_1\le T_{1,ij}\le U_1\right]\left(pt_{1,j}+P_{1,FIFO}\sum_{\substack{r=1\\ r\neq j}}^{J}\left(\lambda_{1,r}/\lambda_1'\right)s_{1,jr}\right) \quad (5)$$

In Equation (5), $E(ST_{1,ij})$ depends on the probability $P_{1,FIFO}$. Therefore, before calculating the expected service time of the i^{th} job of type j at the first stage ($E(ST_{1,ij})$), it is necessary to calculate the probability $P_{1,FIFO}$ in advance. For further details about the mathematical proof in the probability $P_{1,FIFO}$, see Yang *et al*. (2006).

The calculation of the expected waiting time

Considering jobs arrive at the first stage with a Poisson rate $\lambda_1=\sum_{j=1}^{J}\lambda_{1,j}$ and will be served by any one of k_1 machines, each of which has the general distribution. Therefore, the queuing system at the first stage is the model $M/G/K$. One job arrives at the first stage, waiting would be necessary for an arriving job while there are ns_1 jobs in the system, $ns_1\ge k_1$. Let Ns_1 be the number of jobs present at the first stage, and the approximation for $p_{ns_1,1}$ $(=\Pr[Ns_1=ns_1])$ at the first stage is equal to $[(\lambda_1 E(ST_1))^{ns_1}/k_1!(k_1)^{ns_1-k_1}]p_{0,1}$, where $p_{0,1}=[\sum_{n=0}^{k_1}[\lambda_1 E\left(ST_1\right)^n]/n!+[\lambda_1 E\left(ST_1\right)^{k_1}]/((1-\rho_1)k_1!)]^{-1}$ and

$ns_1 \geq k_1$. Since the inter-arrival time of product type *j* job at the first stage is independently and exponentially distributed with the parameter $\lambda_{1,j}$, and then the probability of the i^{th} job of type *j* would arrive at the first stage in the horizon $[L_1, U_1]$ is equal to $\Pr[L_1 \leq T_{1,ij} \leq U_1]$, where $T_{1,ij}$ is the random variable that the time until the the i^{th} job of type *j* arriving at the first stage, and the probability density function of $T_{1,ij}$ is the gamma distribution with parameters $\lambda_{1,j}$ and *i*.

Therefore, the waiting time of a new arriving job relates to the expected service time of some specific number of jobs already arrived before the new arriving job. Supposing that a new job arrives at the first stage and there are $ns_1 = k_1$ jobs present in the system, the new arriving job have to wait in the queue until one job completed its processing. The service time of the product type *j* job would be equal to its processing time with at least one job with a probability equal to $(\lambda_{1,j} / \lambda_1)(1 - P_{1,FIFO})$ and the service time of the product type *j* job would be equal to its processing time plus its setup time and a probability equal to $(\lambda_{1,j} / \lambda_1) P_{1,FIFO} \left(\lambda_{1,r} / \lambda_1'\right)$, where $j = 1, 2, \ldots J$, $r = 1, 2, \ldots J$ and $r \neq j$. Thus, for a new arriving job of the i^{th} job of type *j*, the expected waiting time until one job completed its processing at the first stage can be calculated as Equation (6).

Supposing that a new job arrives at the first stage and there are $ns_1 = k_1 + 1$ jobs present in the system, the new arriving job have to wait in the queue until two jobs completed their processing. Before starting a job's processing, there are two setup cases, "setup" and "no setup". Therefore, the setup cases for two jobs list as follows: "setup, setup", "setup, no setup", "no setup, setup", and "no setup, no setup". Thus, for a new arriving job of the i^{th} job of type *j*, the expected waiting time until two jobs completed their processing at the first stage can be calculated as Equation (7).

$$
\begin{aligned}
EW_{1,ij,one} &= \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1,1} \sum_{j=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} P_{1,FIFO} \left[pt_{1,j} + \sum_{\substack{r=1 \\ r\ne j}}^{J} \frac{\lambda_{1,r}}{\lambda_1'} s_{1,jr} \right] + \\
&\quad \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1,1} \sum_{j=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \left(1 - P_{1,FIFO}\right) pt_{1,j} \\
&= \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1,1} \sum_{j=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \left[pt_{1,j} + P_{1,FIFO} \sum_{\substack{r=1 \\ r\ne j}}^{J} \frac{\lambda_{1,r}}{\lambda_1'} s_{1,jr} \right]
\end{aligned}
\tag{6}
$$

$$
\begin{aligned}
&EW_{1,ij,two} \\
&= \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+1,1} \sum_{j=1}^{J} \sum_{j'=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \frac{\lambda_{1,j'}}{\lambda_1} P_{1,FIFO} P'_{1,FIFO} \left[pt_{1,j} + \sum_{\substack{r=1 \\ r\ne j}}^{J} \frac{\lambda_{1,r}}{\lambda_1'} s_{1,jr} + pt_{1,j'} + \sum_{\substack{r=1 \\ r\ne j'}}^{J} \frac{\lambda_{1,r'}}{\lambda_1''} s_{1,j'r} \right] + \\
&\quad \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+1,1} \sum_{j=1}^{J} \sum_{j'=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \frac{\lambda_{1,j'}}{\lambda_1} P_{1,FIFO} \left(1 - P'_{1,FIFO}\right) \left[pt_{1,j} + \sum_{\substack{r=1 \\ r\ne j}}^{J} \frac{\lambda_{1,r}}{\lambda_1'} s_{1,jr} + pt_{1,j'} \right] + \\
&\quad \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+1,1} \sum_{j=1}^{J} \sum_{j'=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \frac{\lambda_{1,j'}}{\lambda_1} \left(1 - P_{1,FIFO}\right) P'_{1,FIFO} \left[pt_{1,j} + pt_{1,j'} + \sum_{\substack{r=1 \\ r\ne j'}}^{J} \frac{\lambda_{1,r'}}{\lambda_1''} s_{1,j'r} \right] + \\
&\quad \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+1,1} \sum_{j=1}^{J} \sum_{j'=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \frac{\lambda_{1,j'}}{\lambda_1} \left(1 - P_{1,FIFO}\right)\left(1 - P'_{1,FIFO}\right) \left[pt_{1,j} + pt_{1,j'} \right] \\
&= \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+1,1} \sum_{j=1}^{J} \sum_{j'=1}^{J} \frac{\lambda_{1,j}}{\lambda_1} \frac{\lambda_{1,j'}}{\lambda_1} \left[pt_{1,j} + P_{1,FIFO} \sum_{\substack{r=1 \\ r\ne j}}^{J} \frac{\lambda_{1,r}}{\lambda_1'} s_{1,jr} + pt_{1,j'} + P'_{1,FIFO} \sum_{\substack{r=1 \\ r\ne j'}}^{J} \frac{\lambda_{1,r'}}{\lambda_1''} s_{1,j'r} \right]
\end{aligned}
\tag{7}
$$

Furthermore, for a new arriving job of the i^{th} job of type j, the expected waiting time until θ jobs completed their processing at the first stage can be calculated as Equation (8).

$$
\begin{aligned}
EW_{1,ij,\theta} &= \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+(\theta-1),1} \sum_{j^{(1)}=1}^{J} \cdots \sum_{j^{(\theta)}=1}^{J} \frac{\lambda_{1,j^{(1)}}}{\lambda_1} \times \cdots \times \frac{\lambda_{1,j^{(\theta)}}}{\lambda_1} \left[EST_{1,j^{(1)}} + \cdots + EST_{1,j^{(\theta)}} \right] \\
&= \Pr\left[L_1 \le T_{1,ij} \le U_1\right] p_{k_1+(\theta-1),1} \sum_{j^{(1)}=1}^{J} \cdots \sum_{j^{(\theta)}=1}^{J} \frac{\lambda_{1,j^{(1)}}}{\lambda_1} \times \cdots \times \frac{\lambda_{1,j^{(\theta)}}}{\lambda_1} \sum_{\nu=1}^{\theta} EST_{1,j^{(\nu)}}
\end{aligned}
\tag{8}
$$

where $EST_{1j^{(\nu)}} = pt_{1,j^{(\nu)}} + P_{1,FIFO}^{(\nu)} \sum_{r^{(\nu)}=1, r^{(\nu)}\ne j^{(\nu)}}^{J} \left(\lambda_{1,r^{(\nu)}} \big/ \lambda_1^{(\nu)} \right) s_{1,jr^{(\nu)}}$, $\lambda_1^{(\nu)} = \sum_{r^{(\nu)}=1, r^{(\nu)}\ne j^{(\nu)}}^{J} \lambda_{1,r^{(\nu)}}$, $\theta = 1,2,\ldots,\infty$, $j^{(\theta)} = 1,2,\ldots,J$, $r^{(\theta)} = 1,2,\ldots,J$, and $r^{(\theta)} \ne j^{(\theta)}$. Therefore, the expected waiting time of the i^{th} job of type j spent at the first stage can be calculated as Equation (9).

$$E\left(W_{1,ij}\right)=\sum_{\theta=1}^{\infty}EW_{1,ij,\theta}=\Pr\left[L_1\le T_{1,ij}\le U_1\right]\left[\sum_{\theta=1}^{\infty}p_{k_1+(\theta-1),1}\sum_{j^{(1)}=1}^{J}\cdots\sum_{j^{(\theta)}=1}^{J}\frac{\lambda_{1,j^{(1)}}}{\lambda_1}\times\cdots\times\frac{\lambda_{1,j^{(\theta)}}}{\lambda_1}\sum_{\nu=1}^{\theta}EST_{1,j^{(\nu)}}\right] \tag{9}$$

To facilitate the calculation of the expected waiting time of the i^{th} job of type j, Equation (8) need to be reformulated and can be shown as Equation (10).

$$EW_{1,ij,\theta}=\begin{cases}EW_{1,ij,\theta}, & \theta=1\\ \rho_1^{\theta-1}EW_{1,ij,one}+\rho_1\sum_{j=1}^{J}\frac{\lambda_{1,j}}{\lambda_1}EW_{1,ij,\theta-1}, & \theta=2,3,\cdots,\infty\end{cases} \tag{10}$$

2.2 The estimated arrival rates of jobs at the second stage

In order to calculate the completion time of jobs at the second stage, the arrival rates of jobs at the second have to define in advance. Assume that the stages of the flexible flow shop are all independent. Let $N_{2,j}(t)$ be the number of arriving jobs of product type j by time t at the second stage. Suppose that $\{N_{2,j}\left(t\right),t\ge 0\}$ is a Poisson process with arrival rate $\lambda_{2,j}$, $j=1,2,\ldots,J$. The total arrival rate at the second stage λ_2 is equal to the sum of $\lambda_{2,j}$, $\sum_{j=1}^{J}\lambda_{2,j}$.

Since $E(C_{1,ij})$ indicates the expected value of the completion time of the i^{th} job of product type j at the first stage and the moving time of jobs between each stage is assumed to be equal to zero, $E(C_{1,ij})$ can represent the arrival time of the i^{th} job of type j at the second stage. Therefore, the inter-arrival time between the i^{th} job of type j and the $(i+1)^{\text{th}}$ job of type j at the second stage can be shown as Equation (11).

$$
\begin{aligned}
d_{2,i,i+1,j} &= E\left(C_{1,(1+1)j}\right) - E\left(C_{1,ij}\right) \\
&= \frac{i+1}{N_j+1}\left(U_1 - L_1\right) - \frac{i}{N_j+1}\left(U_1 - L_1\right) + \left\{\Pr\left[L_1 \le T_{1,(i+1)j} \le U_1\right] - \Pr\Pr\left[L_1 \le T_{1,ij} \le U_1\right]\right\} \times \\
&\quad \left(pt_{1,j} + P_{1,FIFO}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_{1,r}}{\lambda_1'}s_{jr}\right) + \left\{\Pr\left[L_1 \le T_{1,(i+1)j} \le U_1\right] - \Pr\Pr\left[L_1 \le T_{1,ij} \le U_1\right]\right\}\left(\sum_{\theta=1}^{\infty} EW_{1,ij,\theta}\right) \qquad (11) \\
&= \frac{1}{N_j+1}\left(U_1 - L_1\right) + \left\{\Pr\left[L_1 \le T_{1,(i+1)j} \le U_1\right] - \Pr\Pr\left[L_1 \le T_{1,ij} \le U_1\right]\right\} \times \\
&\quad \left[pt_{1,j} + P_{1,FIFO}\sum_{\substack{r=1\\ r\neq j}}^{J}\frac{\lambda_{1,r}}{\lambda_1'}s_{jr} + \sum_{\theta=1}^{\infty} EW_{1,ij,\theta}\right]
\end{aligned}
$$

Thus, the arrival rate of the product type *j* at the second stage is equal to the inverse of the mean of the inter-arrival time for the product type *j* and can be shown as Equation (12).

$$
\lambda_{2,j} = \left[\begin{array}{l} \dfrac{U_1 - L_1}{N_j+1} + \left\{\Pr\left[L_1 \le T_{1,N_j j} \le U_1\right] - \Pr\left[L_1 \le T_{1,1j} \le U_1\right]\right\} \times \\ \left[\left[pt_{1,j} + P_{1,FIFO}\sum_{\substack{r=1\\ r\neq j}}^{J}\dfrac{\lambda_{1,r}}{\lambda_1'}s_{jr} + EW_{1,ij,one} + \sum_{\theta=2}^{\infty}\left(\rho_1^{\theta-1}EW_{1,ij,one} + \rho_1\sum_{j=1}^{J}\dfrac{\lambda_{1,j}}{\lambda_1}EW_{1,ij,\theta-1}\right)\right]\left(N_j - 1\right)^{-1}\right] \end{array}\right]^{-1} \qquad (12)
$$

2.3 The estimated utilization rates of machines at the second stage

The estimated completion time of jobs at second stage can be determined with the sum of the expected arriving time, the expected waiting time, and the expected service time of jobs at the second stage. The expected waiting time of job relates to the utilization rates of machines, which should be corrected due to the idleness of machine. The idle time depends on the number of jobs in the system at specific time, in which the idle time can be described by the birth-and-death process. Suppose that the number of jobs present in the system at the second stage are smaller than the number of machines, the idle time can be estimated with the time until the next jobs arrived or the time until jobs completed their processing. The time until the next jobs arrived is the inter-arrival of jobs at the second stage, which is the exponential distribution with the parameter λ_2. The time until jobs completed their processing is the service time of jobs.

Assume that there are ns_2 jobs present in the system at the second stage and $ns_2 \le k_2$, where k_2 is the number of machine at the second stage. The process enter this state from state $(ns_2 - 1)$ or state $(ns_2 + 1)$. Thus the mean entering rate must be $(\lambda_2 p_{ns_2-1,2} + \mu_2 p_{ns_2+1,2})$, where $p_{ns_2,2}$ is the probability that there are ns_2 jobs present in the system at the second, and $\mu_2 = 1/E(ST_2)$. In a long period of time τ, and thus state ns_2 have to occur $\tau(\lambda_2 p_{ns_2-1,2} + \mu_2 p_{ns_2+1,2})$ times in time τ, where time τ is the available capacity for each machine at the second stage. Therefore, the total idle time can be calculated as Equation (13).

$$EId_2 = \tau \left\{ \mu_2 p_{1,2} \frac{k_2}{\lambda_2} + \left[\frac{1}{\lambda_2} + E(ST_2) \right] \sum_{i=1}^{k_2-1} (k_2 - i)(\lambda_2 p_{i-1,2} + \mu_2 p_{i+1,2}) \right\} \tag{13}$$

From Equation (13), the machine utilization rate at second stage can be obtained and is expressed as Equation (14).

$$\widehat{UR_2} = \frac{ETST_2}{ETST_2 + EId_{2,k}} \tag{14}$$

where $ETST_2$ is the expected total service time of all jobs at the second stage.

2.4 The estimated completion time of jobs at the second stage

In the similar composition defined in Equation (1), the estimated completion time of the i^{th} job of product type j at the second stage is equal to the sum of the arrival time of the i^{th} job of type j at the second stage, the average waiting time for jobs in queue at the second stage, and the average service time for jobs of the product type j at the second stage, which can be shown as Equation (15).

$$E(C_{2,ij}) = E(Y_{2,[i]j}) + E(W_{2,ij}) + E(ST_{2,ij}) \tag{15}$$

where $E(C_{2,ij})$ is the expected completion time of the i^{th} job of type j at the second stage, $E(Y_{2,[i]j})$ is the expected arrival time of the i^{th} job of type j at the second stage, $E(W_{2,ij})$ is the waiting time of the i^{th} job of type j in queue at the second stage, and $E(ST_{2,ij})$ is the expected service time for jobs of the product type j at the second stage.

Suppose that the moving time of jobs between the first stage and the second is equal to zero. Therefore, the expected arrival time of the i^{th} job of type j at the second stage ($E(Y_{2,[i]j})$) is equal to the expected value of the completion time of the i^{th} job of type j at the first stage ($E(C_{1,ij})$). Thus, the estimated completion time of the i^{th} jobs of the product type j at the second stage can be reformulated as Equation (16).

$$\begin{aligned} E\left(C_{2,ij}\right) &= E\left(Y_{2,[i]j}\right) + E\left(W_{2,ij}\right) + E\left(ST_{2,ij}\right) \\ &= E\left(Y_{1,[i]j}\right) + E\left(W_{1,ij}\right) + E\left(ST_{1,ij}\right) + E\left(W_{2,ij}\right) + E\left(ST_{2,ij}\right) \\ &= E\left(Y_{1,[i]j}\right) + \sum_{t=1}^{2} E\left(W_{t,ij}\right) + \sum_{t=1}^{2} E\left(ST_{t,ij}\right) \end{aligned} \tag{16}$$

The calculations of $E(W_{2,ij})$ and $E(ST_{2,ij})$ are presented below. At the second stage, jobs with different product types arrive dynamically in the horizon $[L_2, U_2]$. Therefore, L_2 can be defined as the expected value of the minimum completion time of jobs at the first stage, and U_2 can be defined as the expected value of the maximum completion time of jobs at the first stage. L_2 and U_2 are expressed in Equation (17).

$$\begin{aligned} L_2 &= E\left(Y_{1,[1]}\right) + E\left(W_{1,q}\right) + E\left(ST_1\right) \\ U_2 &= E\left(Y_{1,[N]}\right) + E\left(W_{1,q}\right) + E\left(ST_1\right) \end{aligned} \tag{17}$$

where $E(Y_{1,[1]})$ is the expected arrival time of the first job at the first stage, $E(Y_{1,[N]})$ is the expected arrival time of the last job at the first stage, $E(W_{1,q})$ is the expected waiting time for jobs in queue at the first stage, $E(ST_1)$ is the expected service time of jobs at the first stage, and $N = \sum_{j=1}^{J} N_j$. Since the inter-arrival time of job for the product type j at the first stage is independently and exponentially distributed with the parameter $\lambda_{1,j}$, $j = 1, 2, \ldots, J$, the arrival time of jobs in the horizon at the first stage should be uniformly distributed over $[L_1, U_1]$. Therefore, $E(Y_{1,[1]}) = (U_1 - L_1) / (N + 1)$ and

$E(Y_{1,[N]}) = [(U_1 - L_1)N]/(N+1)$. The expected waiting time for jobs in queue at the first stage $E(W_{1,q})$ can be calculated as $\sum_{j=1}^{J}\sum_{i=1}^{N_j} E(W_{1,ij})/N$. The expected service time of jobs at the first stag $E(ST_1)$ is equal to $\sum_{j=1}^{J}\sum_{i=1}^{N_j} E(ST_{1,ij})/N$. According to L_2 and U_2, $E(W_{2,ij})$ can be calculate in the same way of Equation (9) and $E(ST_{2,ij})$ can be computed in the same way of Equation (5).

3. COMPUTATIONAL RESULTS

To evaluate the performance of the proposed probabilistic model in this paper, a simulation model is built, in which jobs are dispatched to machines according to the FIFO rule. In the simulation model, there are two process stages and each stage is made up of two parallel machines, and the inter-arrival time of jobs is independently and exponentially distributed. With the simulation model, the arrival rate for each product type job at each stage and the lead time for each product type job at each stage can be collected after the simulation runs, which are used to be compared with the results calculated by the corresponding probabilistic models to evaluate the accuracy of the proposed probabilistic models.

An experimental design is conducted by varying three control parameters at various levels, the expected machine utilization rates at the both stages ($\rho = \lambda E(ST)/K$) and the total arrival rate at the first stage (λ_1). That is, an experiment includes three factors, two combinations of the expected machine utilization rate: ($\rho_1 = 0.90$, $\rho_2 = 0.70$) and ($\rho_1 = 0.70$, $\rho_2 = 0.90$), and two levels of the total arrival rate at the first stage (λ_1):1 (1 job arriving in 60 seconds) and 0.5 (0.5 jobs arriving in 60 seconds). Therefore, there would be 4 combinations. Note that jobs of eight different product types ($J = 8$) will arrive dynamically for each combination and the simulation results would be collected after 1,0000 independent simulation runs. For each combination, the simulation models include the matrix of the sequence-dependent setup time for switching product types on a machine at the first stage and the second stage, which are shown below as $\mathbf{ST}_1$ and $\mathbf{ST}_2$.

$$\mathbf{ST}_1 = \begin{bmatrix} 0 & 3 & 6 & 9 & 12 & 9 & 12 & 3 \\ 18 & 0 & 12 & 15 & 15 & 6 & 9 & 6 \\ 12 & 15 & 0 & 18 & 9 & 6 & 12 & 3 \\ 3 & 6 & 9 & 0 & 9 & 6 & 3 & 6 \\ 3 & 9 & 18 & 9 & 0 & 15 & 9 & 12 \\ 6 & 15 & 18 & 6 & 9 & 0 & 3 & 6 \\ 9 & 18 & 15 & 12 & 15 & 12 & 0 & 9 \\ 6 & 9 & 12 & 9 & 3 & 15 & 9 & 0 \end{bmatrix} \qquad \mathbf{ST}_2 = \begin{bmatrix} 0 & 5 & 10 & 15 & 20 & 15 & 20 & 5 \\ 30 & 0 & 20 & 15 & 25 & 10 & 15 & 10 \\ 20 & 25 & 0 & 30 & 15 & 10 & 20 & 5 \\ 5 & 10 & 15 & 0 & 15 & 10 & 5 & 10 \\ 5 & 15 & 30 & 15 & 0 & 25 & 15 & 20 \\ 10 & 15 & 30 & 10 & 15 & 0 & 5 & 10 \\ 15 & 30 & 25 & 20 & 25 & 20 & 0 & 15 \\ 10 & 15 & 20 & 15 & 5 & 25 & 15 & 0 \end{bmatrix}$$

Furthermore, the job arrival rate for each product type at the first stage and the job processing time for each product type at each stage are calculated. First, the arrival rate of the product type *j* job at the first stage ($\lambda_{1,j}$) can be derived as $\lambda_{1,j} = (u_{1,j} / \sum_{j=1}^{8} u_{1,j})\lambda_1$, where $u_{1,j}$ is the random number (beta distribution with parameters $\alpha = 0.65$ and $\beta = 0.35$) at the first stage for $j = 1, 2, \ldots, 8$. Second, the expected service time of jobs at the t^{th} stage can be derived as Equation (18).

$$\begin{aligned} E(ST_t) &= \sum_{j=1}^{8} \sum_{i=1}^{N_j} E(ST_{t,ij}) \Big/ \sum_{j=1}^{8} N_j \\ &= \left(\sum_{j=1}^{8} N_j \right)^{-1} \sum_{j=1}^{8} \sum_{i=1}^{N_j} \Pr\left[L_t \le T_{t,ij} \le U_t \right] \left(pt_{t,j} + P_{t,FIFO} \sum_{\substack{r=1 \\ r \ne j}}^{8} \left(\lambda_{t,r} / \lambda_t' \right) s_{t,jr} \right) \end{aligned} \tag{18}$$

then

$$\begin{aligned} \left(\sum_{j=1}^{8} N_j \right)^{-1} \sum_{j=1}^{8} \sum_{i=1}^{N_j} \Pr\left[L_t \le T_{t,ij} \le U_t \right] pt_{t,j} &= E(ST_t) - \\ & \left(\sum_{j=1}^{8} N_j \right)^{-1} \sum_{j=1}^{8} \sum_{i=1}^{N_j} \Pr\left[L_t \le T_{t,ij} \le U_t \right] P_{t,FIFO} \sum_{\substack{r=1 \\ r \ne j}}^{8} \left(\lambda_{t,r} / \lambda_t' \right) s_{t,jr} \end{aligned} \tag{19}$$

where the expected service time of jobs at the t^{th} stage is given by $E(ST_t) = (\rho_t k_t) / \lambda_t$ and $t = 1, 2$. In order to solve the processing time for each product type by Equation (18), we assume that the ratio of processing time for two specific product types is set to be fixed constant. Note that “second” is the units of processing time and setup time.

Accuracy analysis of the probabilistic model for the job arrival rate

To analyze the accuracy of the probabilistic model on estimating the job arrival rate, the error percentage of estimated job arrival rate is defined by Equation (20).

$$E_{arrival\ \text{rate}} = \frac{\left|\text{ArrivalRate\_E} - \text{ArrivalRate\_S}\right|}{\text{ArrivalRate\_S}} \times 100\% \tag{20}$$

where ArrivalRate_E is the job arrival rate estimated by the probabilistic model and ArrivalRate_S is the job arrival rate determined by simulation model. The values of $E_{arrival\ rate}$ are shown in Table 1. When the total arrival rate at the first stage is increasing, the error percentage of estimated job arrival rate decreases. This means that jobs arrive at the system to be highly concentrated, which has to be contributive to reduce the error percentage of estimated job arrival rate.

Table 1. Computational results of job arrival rate for various expected stage machine utilization rates, total arrival rates at the first stage, and time horizons at the first stage

Product type	(1) $\lambda_1 = 1$, $\rho_1 = 0.90$, and $\rho_2 = 0.70$						(2) $\lambda_1 = 1$, $\rho_1 = 0.70$, and $\rho_2 = 0.90$					
	ArrivalRate_E		ArrivalRate_S		$E_{arrival\ rate}$		ArrivalRate_E		ArrivalRate_S		$E_{arrival\ rate}$	
	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage
1	0.002083	0.002083	0.001987	0.001986	4.831404	4.884189	0.002708	0.002708	0.002717	0.002675	0.331248	1.233645
2	0.001667	0.001667	0.001578	0.001578	5.640051	5.640051	0.002153	0.002153	0.002097	0.002068	2.670482	4.110251
3	0.002500	0.002500	0.002352	0.002350	6.292517	6.382979	0.002639	0.002639	0.002610	0.002570	1.111111	2.684825
4	0.002153	0.002153	0.002048	0.002048	5.126953	5.126953	0.001736	0.001736	0.001703	0.001681	1.937757	3.271862
5	0.002222	0.002222	0.002137	0.002136	3.977539	4.026217	0.002569	0.002569	0.002503	0.002466	2.636836	4.176805
6	0.001875	0.001875	0.001817	0.001816	3.192075	3.248899	0.001458	0.001458	0.001458	0.001440	0.000000	1.250000
7	0.002708	0.002708	0.002619	0.002618	3.398244	3.437739	0.001944	0.001944	0.001933	0.001907	0.569064	1.940220
8	0.002292	0.002292	0.002199	0.002198	4.229195	4.276615	0.002361	0.002361	0.002300	0.002268	2.652174	4.100529
Product type	(3) $\lambda_1 = 0.5$, $\rho_1 = 0.90$, and $\rho_2 = 0.70$						(4) $\lambda_1 = 0.5$, $\rho_1 = 0.70$, and $\rho_2 = 0.90$					
	ArrivalRate_E		ArrivalRate_S		$E_{arrival\ rate}$		ArrivalRate_E		ArrivalRate_S		$E_{arrival\ rate}$	
	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage	2th stage	3th stage
1	0.001181	0.001181	0.001058	0.001057	11.625709	11.731315	0.001181	0.001181	0.001173	0.001150	0.682012	2.695652
2	0.001111	0.001111	0.001025	0.001024	8.390244	8.496094	0.001250	0.001250	0.001214	0.001191	2.965404	4.953820
3	0.000972	0.000972	0.000888	0.000888	9.459459	9.459459	0.001389	0.001389	0.001376	0.001349	0.944767	2.965159
4	0.001250	0.001250	0.001150	0.001149	8.695652	8.790252	0.001181	0.001181	0.001159	0.001136	1.898188	3.961268
5	0.001111	0.001111	0.001013	0.001013	9.674235	9.674235	0.001042	0.001042	0.001041	0.001023	0.096061	1.857283
6	0.000833	0.000833	0.000726	0.000726	14.738292	14.738292	0.001111	0.001111	0.001084	0.001065	2.490775	4.319249
7	0.001389	0.001389	0.001277	0.001275	8.770556	8.941176	0.000903	0.000903	0.000870	0.000855	3.793103	5.614035
8	0.001250	0.001250	0.001137	0.001136	9.938434	10.035211	0.001111	0.001111	0.001088	0.001068	2.113971	4.026217

The combination of $\rho_1 = 0.90$ and $\rho_2 = 0.70$ forms the expected machine utilization rate of the stages, which means that the first stage is the bottleneck. On the other hand, the combination of $\rho_1 = 0.70$ and $\rho_2 = 0.90$ indicates that the second stage is the bottleneck. From the bottleneck to the nonbottleneck, the behavior of jobs arriving at the system has impacts on the waiting time of jobs at the bottleneck to result in the increasing error percentage of estimated job arrival rate. From the nonbottleneck to the bottleneck, the error percentage of estimated job arrival rate is decreasing, which is the results of the slight influence on the waiting time of jobs at the nonbottleneck.

The higher error percentage of estimated job arrival rate occurring for the combination of $\rho_1 = 0.90$ and $\rho_2 = 0.70$ relates to the smaller time horizon at the first stage. The lower error percentage of estimated job arrival rate occurring for the combination of $\rho_1 = 0.70$, and $\rho_2 = 0.90$ relates to the longer time horizon at the first stage. The overall mean of the error percentages of estimated job arrival rate is 4.9519 %. In general, the probabilistic model can estimate the job arrival rate accurately.

Accuracy analysis of the probabilistic model for the lead time of the product type

The accuracy on the estimated stage completion time of jobs is evaluated in this paper. However, the number of jobs generated by the probabilistic model and the simulation model are not the same. Therefore, the lead time for each product type is adopted instead to evaluate the accuracy on the estimated stage completion time of jobs, in which lead time of job is the period of time between the arriving of job at one stage and the completion of job at that stage.

Table 2 displays the lead time at the first stage generated by the probabilistic model and the simulation for various expected stage machine utilization rates, total arrival rates at the first stage, and time horizons at the first stage. Table 3 displays the lead time at the second stage generated by the probabilistic model and the simulation for various expected stage machine utilization rates, total arrival rates at the first stage, and time horizons at the first stage. To analyze the accuracy of the probabilistic model on estimating the lead time of the product type, the error percentage of estimated the lead time is defined by Equation (21). The mean error percentage for the service time is 9.2586% at the first stage and at the second stage 6.8007%, which indicate that the proposed model can estimate the stage service time accurately. However, for the waiting time estimation, the proposed model can not restrict the estimation error in an accepted range, which leads to the estimation error of job lead time. Note that error percentage of estimated waiting time increasing at the second stage, which requires future works to improve the probabilistic model on lead time estimation.

$$E_{lead\ time} = \frac{\left|\text{LeadTime\_E} - \text{LeadTime\_S}\right|}{\text{LeadTime\_S}} \times 100\% \tag{21}$$

where LeadTime_E and LeadTime_S respectively represent the lead time done by the probabilistic and simulation model.

Table 2. Computational results of lead time and lead time components at the first stage for various expected stage machine utilization rates, total arrival rate, and time horizons

Product type	(1) $\lambda_1 = 1$, $\rho_1 = 0.90$, and $\rho_2 = 0.70$						(2) $\lambda_1 = 1$, $\rho_1 = 0.70$, and $\rho_2 = 0.90$					
	Lead time		Waiting time		Service time		Lead time		Waiting time		Service time	
	E	S	E	S	E	S	E	S	E	S	E	S
1	1038.5838	607.4133	921.0752	478.5458	117.5085	128.8675	203.8904	138.8136	162.4665	96.4923	41.4239	42.3213
2	1068.4953	642.6080	919.6893	474.7983	148.8060	167.8097	260.3990	204.5555	161.6665	95.9087	98.7325	108.6468
3	1012.6846	576.6538	921.6613	479.7986	91.0232	96.8552	214.2058	150.1459	162.3073	96.7048	51.8985	53.4411
4	1032.3448	596.6645	921.3457	475.7970	110.9991	120.8674	293.1178	244.8312	161.3566	98.2210	131.7612	146.6102
5	1030.8334	595.8200	922.1037	478.8495	108.7297	116.9706	221.3461	158.7718	162.1441	96.7606	59.2020	62.0113
6	1053.4910	621.0111	921.0382	475.3162	132.4528	145.6949	318.8210	274.2132	161.0549	95.9254	157.7660	178.2878
7	999.0553	554.3153	922.7272	476.3996	76.3281	77.9156	278.2930	225.1498	161.9128	97.1290	116.3802	128.0208
8	1024.8022	589.0092	921.9565	478.4734	102.8457	110.5358	238.7528	177.7016	162.0091	95.4862	76.7437	82.2154
Product type	(3) $\lambda_1 = 0.5$, $\rho_1 = 0.90$, and $\rho_2 = 0.70$						(4) $\lambda_1 = 0.5$, $\rho_1 = 0.70$, and $\rho_2 = 0.90$					
	Lead time		Waiting time		Service time		Lead time		Waiting time		Service time	
	E	S	E	S	E	S	E	S	E	S	E	S
1	2060.5653	1374.3754	1840.5274	1122.5289	220.0379	251.8464	480.3514	434.5135	324.3160	260.2559	156.0354	174.2577
2	2079.1515	1367.8869	1845.5926	1107.7870	233.5590	260.0999	467.1016	425.8445	322.9679	262.8850	144.1337	162.9595
3	2140.5960	1445.0049	1841.5144	1102.5306	299.0816	342.4743	409.5483	352.6175	324.6491	260.3733	84.8991	92.2442
4	2016.8630	1310.7849	1846.0464	1121.9562	170.8166	188.8287	481.0889	438.2564	323.9062	261.4674	157.1827	176.7891
5	2080.6122	1375.5720	1844.7358	1112.0370	235.8765	263.5349	532.8713	495.0428	323.6305	257.8498	209.2408	237.1930
6	2193.3981	1519.4678	1833.2808	1087.4534	360.1174	432.0143	511.7087	477.9153	322.7908	262.0959	188.9179	215.8194
7	1961.9260	1253.5021	1847.3305	1130.9547	114.5955	122.5474	585.7644	569.1297	321.3015	260.2376	264.4629	308.8921
8	2025.2026	1334.1322	1844.3018	1132.6063	180.9008	201.5259	511.0011	476.3606	322.9706	262.2711	188.0305	214.0895

Table 3. Computational results of lead time and lead time components at the second stage for various expected stage machine utilization rates, total arrival rates, and time horizons

Product type	(1) $\lambda_1 = 1$, $\rho_1 = 0.90$, and $\rho_2 = 0.70$						(2) $\lambda_1 = 1$, $\rho_1 = 0.70$, and $\rho_2 = 0.90$					
	Lead time		Waiting time		Service time		Lead time		Waiting time		Service time	
	E	S	E	S	E	S	E	S	E	S	E	S
1	120.4962	55.5010	71.5314	2.9666	48.9648	52.5343	446.7822	400.8119	269.7200	216.9978	177.0621	183.8141
2	175.3445	116.2906	71.3534	1.4331	103.9912	114.8574	341.0264	264.0825	269.4583	187.4146	71.5681	76.6679
3	194.6241	136.2260	71.6687	8.1920	122.9554	128.0340	400.5069	344.4213	269.6913	210.2636	130.8156	134.1577
4	122.1908	57.5196	71.5566	3.7733	50.6341	53.7463	347.2065	261.4326	269.1968	175.8182	78.0097	85.6144
5	118.3721	52.8209	71.5809	3.7924	46.7912	49.0285	363.1617	303.1135	269.6616	207.7166	93.5001	95.3969
6	199.0819	140.0514	71.4487	1.9297	127.6333	138.1217	324.3931	230.0159	268.9736	165.2462	55.4195	64.7697
7	143.8381	81.4984	71.7267	11.4600	72.1113	70.0384	390.8530	313.8723	269.3365	182.2453	121.5164	131.6270
8	145.8710	82.8043	71.6041	5.0759	74.2669	77.7284	322.8156	254.3483	269.5658	200.1602	53.2499	54.1881
Product type	(3) $\lambda_1 = 0.5$, $\rho_1 = 0.90$, and $\rho_2 = 0.70$						(4) $\lambda_1 = 0.5$, $\rho_1 = 0.70$, and $\rho_2 = 0.90$					
	Lead time		Waiting time		Service time		Lead time		Waiting time		Service time	
	E	S	E	S	E	S	E	S	E	S	E	S
1	244.7774	123.7874	134.0633	4.2447	110.7141	119.5428	736.1560	525.8569	499.6663	270.0701	236.4897	255.7868
2	319.7225	206.0984	133.9596	4.4473	185.7629	201.6510	659.5729	444.9514	499.8143	273.8282	159.7586	171.1233
3	266.5045	149.8271	133.7266	2.3848	132.7779	147.4422	687.1506	503.1895	500.0822	305.4555	187.0684	197.7340
4	260.3913	145.7634	134.1600	10.4622	126.2313	135.3012	755.8623	544.9693	499.6663	267.0032	256.1960	277.9661
5	320.1895	205.6077	133.9596	4.0509	186.2299	201.5568	645.5477	406.4167	499.3358	245.7626	146.2118	160.6542
6	264.8703	148.5815	133.4504	1.6195	131.4199	146.9620	648.5037	411.9025	499.5075	249.1194	148.9962	162.7831
7	354.2787	258.8685	134.3359	26.0346	219.9428	232.8338	785.1067	533.7050	498.9480	216.2196	286.1587	317.4854
8	256.5589	138.6225	134.1600	8.4818	122.3990	130.1407	664.8455	431.7985	499.5075	251.7127	165.3380	180.0858

Table 4. The results of the goodness of fit and Kolmogorov-Smirnov test

Stage	Test	1	2	3	4	5	6	7	8	9	10
1	Goodness of fit	0.78004	0.51222	0.73685	0.36732	0.81025	0.94456	0.80169	0.48022	0.55668	0.51569
	Kolmogorov-Smirnov	0.516	0.641	0.700	0.243	0.848	0.446	0.736	0.907	0.378	0.436
2	Goodness of fit	0.00002	0.01604	0.00000	0.00001	0.00244	0.00000	0.00086	0.00011	0.01122	0.16803
	Kolmogorov-Smirnov	0.051	0.021	0.003	0.001	0.001	0.000	0.019	0.064	0.187	0.404

Table 4 presents the results of the goodness of fit and Kolmogorov-Smirnov test for some simulation runs, in which the null hypothesis that the inter-arrival time of jobs is generated by the simulation at the first stage and the second stage from a population that follows the exponential distribution. Table 4 reveals that the P-value at the first stage is more than the 0.05 level of significance, and then we can conclude at the 0.05 level of significance that the inter-arrival time of jobs by the simulation at the first stage from a population that follows the exponential distribution. However, some P-value at the second stage are less than the 0.05 level of significance, and then he inter-arrival time of jobs by the simulation at the second stage did not come from a population that follows the exponential distribution.

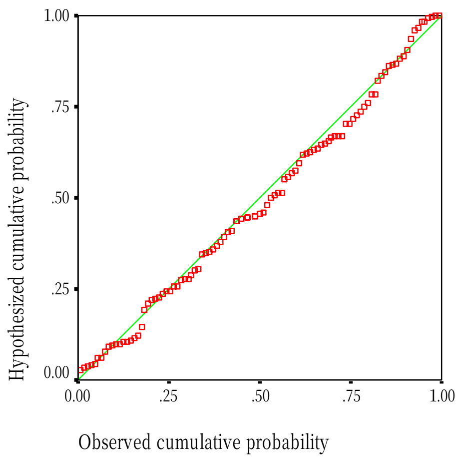

Figure 1. The p-p plots of the inter-arrival time of jobs
by the simulation at the second stage for one simulation run

Therefore, to change the distribution of the inter-arrival time of jobs at the second stage would improve the probabilistic model on lead time estimation. Figure 1 is the p-p plot of the inter-arrival time of jobs by the simulation at the second stage for one simulation run, in which the null hypothesis states that the inter-arrival time follows the Weibull distribution. In Figure 1, the observed data keep close to the 45° line. Therefore, in the future research, we can assume that the inter-arrival time of jobs at the second stage as the Weibull distribution to improve the estimation of lead time by the probabilistic model.

4. CONCLUSIONS

This paper developed the probabilistic model to estimate the job completion time at each stage for the two-stage flexible flow shop problem with dynamic job arrival. To evaluate the performance of the probabilistic model, a simulation model was built and an experimental design was conducted to evaluate the accuracy of the proposed probabilistic model on estimating the stage completion time. Note the lead time for each product type is adopted instead to evaluate the accuracy on the estimated stage completion time of jobs. Computational results show that the proposed probabilistic model for estimating the job arrival rate for each product type at the stages has been shown to be effective. As the total arrival rate at the first stage is increasing, the error percentage of estimated job arrival rate decreases. The overall mean of the error percentages of estimated job arrival rate is 4.9519%. The overall mean of the error percentages of the service time indicates that the stage service time can be estimated accurately. However, for the waiting time estimation, the proposed model can not restrict the estimation error in an accepted range, which leads to the estimation error of job lead time. Future works are required to improve the lead time estimation.

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